Thin-Walled Structures 132 (2018) 316–331

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Cellular buckling from nonlinear mode interaction in unequal-leg angle struts

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ARTICLE INFO

Keywords:
Structural stability
Mode interaction
Snaking
Nonlinear mechanics
Thin-walled structures

ABSTRACT

A variational model based on total potential energy principles that describes the nonlinear mode interaction in thin-walled unequal-leg angle struts under pure axial compression is presented. The formulation, which combines continuous displacement functions and generalized coordinates, leads to the derivation of a system of differential and integral equations that describe the static equilibrium response of the strut. Solving the system of equations using numerical continuation techniques reveals, for the first time, progressive cellular buckling (or *snaking*) represented by a sequence of snap-back instabilities arising from the nonlinear interaction of the weak-axis flexural, strong-axis flexural and torsional buckling modes—the resulting behaviour being highly unstable. For verification purposes, a finite element (FE) model is also devised and the sequential snap-back instabilities are also captured within its framework. Moreover, once an initial geometric perturbation is incorporated within the variational model it compares very well with the FE model.

1. Introduction

The buckling of struts represents one of the most common types of structural instability problem [1]. Thin-walled metallic structural components in compression are well known to suffer from a variety of different elastic buckling phenomena due to the high slenderness ratio at both local and global scales [2–6]. In the current work, a strut under axial compression made from a linearly elastic material with an asymmetric unequal-leg angle cross-section is studied using an analytical approach based on variational principles. Cross-sections with no axis of symmetry are well known to suffer from instabilities that combine flexure and torsion when subjected to compression [7]. For an unequal-leg angle member, the torsional component is represented by the cross-section rotation about the shear centre that lies on neither of the principal axes for bending of the cross-section. Allen and Bulson [8] showed that for this type of cross-section, weak and strong-axis flexural displacements always occur in conjunction with the torsional displacements, leading to a flexural–torsional buckling mode with biaxial bending. Moreover, since unequal-leg angle sections effectively comprise two rectangular plate elements, an alternative approach to describe the torsional component is to consider buckling of the individual plate elements that are pin-jointed at the common longitudinal edge, whereas the opposite edges remain free [8–10]—a point that is discussed in detail later. This approach has been shown to be perfectly applicable in a recent study on equal-leg angle struts [11], where the nonlinear mode interaction between the weak-axis flexural (Euler) buckling and the strong-axis flexural–torsional buckling was studied, recalling that for mono-symmetric cross-sections, weak-axis flexural buckling may occur as an independent mode [8].

In recent decades, the buckling phenomena found in struts and beams made from angle sections have been studied by various researchers through experimental and numerical approaches [12–17]. Popovic et al. [18] conducted a series of compression tests on equal-leg angle struts made from cold-formed steel and found that a number of test specimens underwent an interaction between Euler buckling and the strong-axis flexural–torsional buckling modes. Dinis et al. [10] also found this type of mode interaction using a combination of Generalized Beam Theory and finite element analysis. Although extensive studies have been conducted on equal-leg angle sections, fewer studies have focused specifically on unequal-leg angles [19–21]. Trahair conducted a series of studies [22–24] on unequal-leg angle beams exhibiting biaxial bending and torsion. Young and Chen [25] conducted a series of experiments on lipped unequal-leg angle columns, revealing the flexural–torsional buckling mode with biaxial bending; Liu and Chantel [26] conducted a series of experiments on unequal-leg angle struts under eccentric compression, also revealing the interaction of weak-axis

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https://doi.org/10.1016/j.tws.2018.08.007
Received 11 April 2018; Received in revised form 3 July 2018; Accepted 7 August 2018
0263-8231/

flexural, strong-axis flexural and torsional buckling. It is well-known that when two or more buckling modes occur simultaneously, the post-buckling behaviour can be far more unstable than when they are triggered in isolation [27–31]. As far as the authors are aware, a mathematical model that describes the nonlinear mode interaction of weak-axis flexural, strong-axis flexural and torsional buckling in unequal-leg angle struts has not yet been forthcoming.

"Cellular buckling" [32] or "snaking" [33], represented by snap-back instabilities on equilibrium paths, showing sequential destabilization and restabilization alongside a progressive formation of new peaks and troughs in the deformation profile, has been found in mechanical systems such as in the post-buckling of cylindrical shells [34], stiffened panels [35], thin-walled I-section columns [36] and beams [37]. In a recent study [11], cellular buckling was captured by both the variational model and experiments for equal-leg angle struts exhibiting nonlinear mode interaction. The findings of [10] also revealed snap-back instabilities on the equilibrium path for equal-leg angle struts alongside the switch from one "half-wave" to three "half-waves" in the torsional buckling profiles. It therefore may be expected that cellular buckling also occurs in unequal-leg angle struts. The current article presents the development of a variational model of an unequal-leg angle strut with and without initial geometric imperfections. A system of nonlinear ordinary differential equations subject to boundary and integral conditions is derived and solved using the numerical continuation package AUTO [38]. It is worth noting that, in contrast to [11], a further set of displacement functions needs to be introduced to the system and solved for numerically. Despite the further complexity in the equilibrium equations, this new set of functions is shown to be critically important and leads to a series of distinct features associated with the snap-back instabilities.

For verification purposes, a purely numerical model is devised within the commercial finite element (FE) package ABAQUS [39]. Studies are presented for a series of unequal-leg angle struts with different geometric properties. Snap-back instabilities are captured by both the variational and the FE model; the results from the two models compare excellently both in terms of the mechanical destabilization and the post-buckling deformation. A brief discussion on potential further studies is presented before final conclusions are drawn.

2. Variational model

Consider a thin-walled unequal-leg angle strut of length L that is made from a linear elastic, homogeneous and isotropic material with Young's modulus E and Poisson's ratio ν. The coordinate systems and cross-sectional properties are shown in Fig. 1(a); the leg thickness and widths are denoted as t and h_i respectively where $i = \{1, 2\}$, and it is assumed that $h_1 \geq h_2$ throughout the current article. Note that the origins of both coordinate systems are located at the shear centre of the cross-section $\mathscr{S}$; the vertical distance between the geometric centroid $\mathscr{C}$ and the centreline of each leg is $\bar{x}_i = h_i^2/[2(h_1 + h_2)]$ assuming that $h_i \gg t$, as shown in Fig. 1(a). Note that the coordinate system x_1^{T}–x_2^{T} is obtained by rotating the x_1–x_2 axes through an angle α clockwise, allowing x_1^{T} and x_2^{T} to be parallel to the principal weak and strong axes of bending of the cross-section respectively. The angle of rotation of the principal axes, α, is determined by constructing a Mohr's circle for the second moment of area, thus:

$$\alpha = \frac{1}{2}\left\{\pi - \arctan\left[\frac{6\psi^2}{(1-\psi^2)(1+4\psi+\psi^2)}\right]\right\}, \tag{1}$$

where $\psi = h_2/h_1$. The distance between $\mathscr{S}$ and $\mathscr{C}$ in the x_i^{T} direction, $\bar{x}_i^{\mathrm{T}}$, is thus:

$$\bar{x}_i^{\mathrm{T}} = \frac{h_i}{2(1+\psi)\psi^{i-1}}\{(1+\psi^4)[i-1-(-1)^i\cos^2(\alpha - \arctan\psi^2)]\}^{\frac{1}{2}}. \tag{2}$$

The strut is loaded by an axial force P that is applied at the centroid of the cross-section, as shown in Fig. 1(b), and it is assumed that the force is transferred uniformly across the entire cross-section. The strut is simply-supported by "spherically-pinned" conditions where the cross-section is free to rotate about both principal axes, but constrained against the rotation about the longitudinal axis, at both ends of the strut.

2.1. Modal description

Fig. 1(c) shows the decomposition of the interactive buckling mode where the strut exhibits a weak-axis flexural (global) displacement, a strong-axis flexural (global) displacement and a displacement arising from the torsional rotation. It is worth emphasizing that, since the cross-section is asymmetric, all three displacement components are expected to be triggered simultaneously, leading to the flexural–torsional buckling mode with biaxial bending. Moreover, it was shown in [11] that the torsional displacement could be described by considering each leg as an individual buckled plate element with the common longitudinal edge simply-supported and the opposite edges being free [8]. Bulson [40] showed that, for the current type of rectangular plate, the buckling eigenmode has a linear distribution of displacement across the width of the plate. Hence, the plate buckling displacement profiles essentially resemble the rotation of the entire cross-section about its shear centre, assuming that the angle of rotation ϕ is identical for both plate elements when they buckle simultaneously, as shown in Fig. 1(c). Moreover, Allen and Bulson [8] demonstrated for a cruciform (+) cross-section that the critical buckling load determined from plate buckling theory is more accurate than that obtained from torsional buckling theory. Therefore, the current work considers the torsional component of the interactive buckling mode being a result of buckling of the plate elements that represent the angle legs. For the purposes of clarity, the corresponding displacement profile, as shown by the bottom-right figure in Fig. 1(c), is referred to hereafter as the plate buckling displacement, as opposed to the displacement arising from torsion.

Timoshenko beam theory is assumed, implying that flexural shear strains are included [36,11]. Two generalized coordinates, q_s and q_t, defined as the amplitudes of the degrees of freedom known as "sway" and "tilt", are introduced to describe the weak-axis flexural component [3], as shown in Fig. 1(d). Thus, the weak-axis lateral displacement W_w and cross-section rotation θ_w are given by the following expressions:

$$W_w(z) = q_s L \sin\frac{\pi z}{L},$$
$$\theta_w(z) = -q_t \pi \cos\frac{\pi z}{L}. \tag{3}$$

On the other hand, the strong-axis lateral displacement $W_s(z)$ and cross-section rotation $\theta_s(z)$ are introduced as continuous functions and solved for numerically, which is a departure from previous work [11]. For the present case, the shear strains from the global flexural displacements in the $x_1^{\mathrm{T}}z$ and $x_2^{\mathrm{T}}z$ planes are denoted as $\gamma_{x_1^{\mathrm{T}}z}$ and $\gamma_{x_2^{\mathrm{T}}z}$ respectively, and given by the following expressions:

$$\gamma_{x_1^{\mathrm{T}}z} = \frac{\mathrm{d}W_w}{\mathrm{d}z} + \theta_w = (q_s - q_t)\pi\cos\frac{\pi z}{L},$$
$$\gamma_{x_2^{\mathrm{T}}z} = \frac{\mathrm{d}W_s}{\mathrm{d}z} + \theta_s. \tag{4}$$

Unlike in previous work [11], the functions W_s and θ_s are initially unknown and are included in the minimization of the total potential