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Spectral stochastic isogeometric analysis for static response of FGM plate with material uncertainty

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ABSTRACT

In this study, the nondeterministic structural responses of functionally graded material (FGM) plates under static loads with uncertain material property is investigated. The considered spatially dependent uncertainties are modelled as random fields with Gaussian distribution. A novel spectral stochastic isogeometric analysis (SSIGA) framework is proposed for such uncertainty quantification through the first-order shear deformation theory. Within the SSIGA framework, the non-uniform rational B-spline (NURBS) is adopted for both the geometry modelling of the random fields of the uncertain material properties and random field discretization through the Karhunen-Loève (K-L) expansion. Such new feature provides an effective and practically applicable random field modelling technique, especially for uncertain parameters over complex physical domains. The polynomial chaos expansion (PCE) is employed for estimating the statistical characteristics (e.g., mean and standard deviation) of any concerned structural responses (e.g., displacement and stress). By further implementing various statistical inference techniques, the probability density functions (PDF) and cumulative distribution functions (CDF) of structural responses can be established to determine both serviceability and strength limits of FGM plate. Two numerical examples are thoroughly investigated to illustrate the applicability, effectiveness and efficiency of the proposed computational approach.

1. Introduction

Since the first successful invention of artificial functionally graded materials (FGM) in the 1980's, such advanced composite material has attracted lots of attention from both academic and industrial communities [1–5]. By manipulating the composition and microstructure, the FGM meticulously offers a continuous variation in the mechanical properties of the advanced composite at macroscopic level and adequately combines dissimilar mechanical properties through the gradation process [6]. The benefit of such meticulous manufacturing process is that the mechanical properties of the FGM can be more specifically optimized to suffice various demands inspired from real-life engineering applications [6]. In addition to the classical application of FGM as thermal barrier coatings for shielding aerospace machinery and gas-turbine components [7], the FGM has also been successfully implemented across a wide range of engineering disciplines including automotive, civil, military, biomedical, and electrical etc. [8–12].

Even though the FGM has been extensively developed and implemented in real-life applications, there are still some challenges that are obstructing the engineers to possess complete control of the composite. One inevitable, yet critical, issue has been realized in engineering practice is that the actual manufactured gradation of the FGM is often different with the designed specification due to the inherent uncertainties involved in the intricate fabrication processes [6]. For example, the random variation of the local volume fraction of a ZrO_2 chromium-nickel alloy FGM was observed in [13]. Such common phenomenon indicates that the uniform gradation is only an approximation of the actual microstructure of the FGM and consequently, the evaluation of the performance of the FGM basing on such assumptions is inadequate for practical engineering purposes. Therefore, it is logical and, requisite that nondeterministic types of analysis framework should be developed to assess the performance of the FGM with the consideration of various physically inspired randomness.

In the light of achieving a more appropriate and physically feasible analysis framework for quantitatively assessing the performance of the FGMs, numbers of nondeterministic analysis for FGMs have been developed for diverse types of structural analysis. Yang et al. [14,15] implemented the first-order perturbation approach to investigate the stochastic static response and elastic buckling of FGM plates with material uncertainties. Kitipornchai et al. [16] combined a semi-analytical approach with first-order perturbation approach to study the random vibration of functionally graded laminates subjected to temperature

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variation. Shaker et al. [17] presented a stochastic finite element method (FEM) based free vibration analysis of FGM plate with independent random variables. Jagtap et al. [18] investigated the stochastic nonlinear natural frequency analysis of FGM plate resting on elastic foundation through the first-order perturbation method. All input uncertain parameters are considered as independent random variables and temperature dependent material properties were adopted. Lal et al. [19] implemented the first-order perturbation theory combined with FEM to propose a stochastic FEM for random post-buckling analysis of FGM plates. Also, Lal et al. [20] extended the first-order perturbation theory based stochastic FEM to investigate the post-buckling behaviour of FGM plate subjected to mechanical-thermal loadings with independent material uncertainties. Shegokar and Lal [21] also adopted the first-order perturbation theory to propose a stochastic FEM to investigate the nonlinear bending responses of piezoelectric functionally graded beam subjected to electrical, mechanical and thermal loading conditions simultaneously. Talha and Singh [22] implemented the first-order perturbation theory based stochastic FEM to investigate the first two moments of statistics of the eigen-buckling load factor of FGM plate with temperature dependent material properties. One year later, Talha and Singh [23] extended the first-order perturbation based stochastic FEM to investigate the natural frequencies of FGM plate with temperature dependent material properties. García-Macías et al. [24] proposed a Kriging based metamodel technique for the stochastic free vibration analysis of FGM plates with carbon nanotube reinforcement. Moreover, Wu et al. [25] proposed a stochastic FEM approach for FGM beam involving various uncertainties through the Euler-Bernoulli beam model. By implementing the combination of the first-order perturbation theory and isogeometric analysis (IGA), Hien and Noh [26] proposed a stochastic IGA approach for natural frequency analysis of FGM plates with material randomness. In addition to the stochastic analysis framework for FGMs, Wu et al. [27] proposed a mathematical programming approach based non-stochastic analysis framework for FGM beams involving interval uncertainties.

In this study, a novel computational stochastic analysis framework, namely the spectral stochastic isogeometric analysis (SSIGA) [28], is presented for quantitatively assessing the performance of FGM plates with uncertain material properties under static load. Unlike the abovementioned stochastic analysis methods, the SSIGA approach introduced herein is capable of handling uncertainty analysis involving both spatially independent (i.e., random variables) and dependent (i.e., random fields) uncertain parameters. By implementing the NURBS as the basis functions for the Karhunen-Loève (K-L) expansion, a new, yet practical, random field discretization technique is forged. The benefit of such novel technique is that the random field of uncertain parameter that is acting on complex physical domain can be more systematically and effectively handled. By further implementing the polynomial chaos expansion (PCE) approach, explicit formulations on the first two statistical moments (i.e., means and standard deviations) of any concerned structural responses (i.e., displacements, strain, and stress) of the FGM plate can be expressed. In addition to the estimations on the means and standard deviations of the structural responses, the proposed SSIGA approach can also adequately establish the probability density functions (PDFs) and cumulative distribution functions (CDFs) of the concerned structural responses through the kernel density estimation approach. Consequently, such critical information provides a quantitative measure of performance, which is known as the structural reliability, of FGM plate. Yet, such competence on offering PDFs and CDFs has distinguished the proposed method from the previously developed first-order perturbation based stochastic analysis framework.

In addition to the superiority on the uncertainty quantification, the proposed SSIGA analysis framework also possesses some unique advantages. By developing the stochastic analysis grounding on the success of the deterministic IGA, the introduced SSIGA framework can maintain the exact geometries of both FGM plate and random field acting on the plate between the design model and stochastic analysis model. Such rigor can eliminate the modelling errors that often occur during the model transformation in practical engineering. That is, by establishing a stochastic analysis in Computer-Aided Design (CAD) framework, the consistency of the geometry of an FGM plate in CAD model, deterministic Computer-Aided Engineering (CAE) model, and structural safety assessment model can be exactly maintained. In fact, the proposed SSIGA meticulously combines the CAD, CAE as well as structural safety assessment into a unified framework. This unique feature is extremely important for practically stimulated FGM plates with complex geometries and spatially dependent uncertain system parameters. Without a consistent geometry of an FGM plate between the CAD model and the stochastic analysis model, the outcome of the stochastic analysis would become meaningless which could potentially jeopardize the safety of the FGM plate [29–32]. In such cases, the SSIGA approach can certainly bring ease of modelling with desirable level of accuracy.

The paper is structured as follows. The deterministic static analysis of FGM plate through IGA is presented in Section 2. After that, the proposed SSIGA approach is introduced in Section 3. Subsequently, two numerical examples are investigated in Section 4 to illustrate the effectiveness and efficiency of the SSIGA approach. Finally, the conclusion is drawn in Section 5.

2. Isogeometric static analysis of FGM plate through first-order shear deformation theory

2.1. The material properties of the FGM plate

In this paper, the top and bottom surfaces of the considered ceramic-metal FGM plate with thickness h are assumed to be purely ceramic and metallic respectively. The mid-plane of the plate is in the x-y plane, and the positive z-axis is perpendicularly upward from the mid-plane. For the considered FGM plate, the Young's modulus E, mass density ρ, and Poisson's ratio ν vary along the thickness direction with a power law distribution as following

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{1}$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{2}$$

$$\nu(z) = \nu_m + (\nu_c - \nu_m)\left(\frac{1}{2} + \frac{z}{h}\right)^n \tag{3}$$

where n denotes the gradient index of FGM; z denotes the thickness coordinate within $[-h/2, h/2]$; and the subscripts m and c represent the metal and ceramic constituents, respectively. The material properties of some common FGM components are presented in Table 1.

2.2. First-order shear deformation theory of plate

In this subsection, the formulation of first-order shear deformation theory (FSDT) of plate is briefly presented. Let D be the domain in $\Re^2$ occupied by the mid-plane of the plate. Within the analysis framework of the FSDT [33], the displacement fields $u(x, y, z)$, $v(x, y, z)$, and $w(x, y, z)$ are defined as:

Table 1
Functionally graded material properties.

Property	Aluminium Al	Zirconia-1 ZrO_2-1	Zirconia-2 ZrO_2-2	Alumina Al_2O_3
$E(GPa)$	70	200	151	380
ν	0.3	0.3	0.3	0.3
$\rho(\text{kg}/m^3)$	2707	5700	3000	3800