Expected Scott–Suppes utility representation

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H I G H L I G H T S

• We provide an axiomatic characterization of Expected Scott–Suppes utility representation.
• This can be used in applications that study intransitive indifference under uncertainty.
• Our main result is the natural analog of vNM expected utility theorem for semiorders.
• Our characterization provides an answer to the open problem noted by Fishburn (1968).
• Our representation offers a decision-theoretical interpretation for epsilon equilibrium.

A R T I C L E I N F O

Article history:
Received 7 February 2018
Received in revised form 1 August 2018

Keywords:
Semiorder
Intransitive indifference
Uncertainty
Expected utility
Scott–Suppes representation

A B S T R A C T

We provide an axiomatic characterization for an expected Scott–Suppes utility representation. Such a characterization is the natural analog of the von Neumann–Morgenstern expected utility theorem for semiorders and it is noted as an open problem by Fishburn (1968). Expected Scott–Suppes utility representation is analytically tractable and can be used in applications that study preferences with intransitive indifference under uncertainty. Our representation offers a decision-theoretical interpretation for epsilon equilibrium as well.

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1. Introduction

1.1. Intransitive indifference and semiorders

The standard rationality assumption in economic theory states that individuals have or should have transitive preferences. A common argument to support the transitivity requirement is that, if individuals do not have transitive preferences, then they are subject to money pumps (Fishburn, 1991). Yet, intransitivity of preferences is frequently observed through choices individuals make in real life and in experiments (May, 1954; Tversky, 1969).

Intransitive indifference is a certain type of intransitivity of preferences: an individual can be indifferent between x and y and also y and z, but not necessarily between x and z.

Formal studies of the idea of intransitive indifference go back to as early as the 19th century (Fechner, 1860; Weber, 1834). The Weber–Fechner law states that a small increase in the physical stimulus may not result in a change in perception, which suggests intransitivity of perceptual abilities.

A notable example was given by Jules Henri (Poincaré, 1905): Sometimes we are able to make the distinction between two sensations while we cannot distinguish them from a third sensation. For example, we can easily make the distinction between a weight of 12 g and a weight of 10 g, but we are not able to distinguish each of them from a weight of 11 g. This fact can symbolically be written: \( A = B, B = C, A < C \).

Armstrong (1939, 1948, 1950) has repeatedly questioned the assumption of transitivity of preferences and concluded:\n
\[ x \leq y \leq z \implies x \leq z \]

We would like to thank Itzhak Gilboa, Peter Klibanoff, Asen Kochov, Marciano Siniscalchi, Ran Spiegler, William Thomson, and Kemal Yıldız for helpful comments. We are grateful to two anonymous reviewers for their detailed comments and suggestions. Any remaining errors are ours.

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1 An individual has transitive preferences if whenever the individual thinks that \( x \) is at least as good as \( y \) and \( y \) is at least as good as \( z \), then \( x \) is at least as good as \( z \).

2 This quotation appears in Pirlot and Vincke (1997, p. 19).

3 This quotation appears in Armstrong (1948, p. 3).
That indifference is not transitive is indisputable, and a world in which it were transitive is indeed unthinkable.

Luce (1956) introduced a way to capture the idea of intransitive indifference. He coined the term semiorder by introducing axioms for a binary relation so that it can represent preferences allowing for intransitive indifference. Luce (1956) also illustrated how semorders can be used to capture the concept of just noticeable difference in psychophysics. Since then, semorders have been studied extensively in preference, choice, and utility theory (Aleskerov, Bouyssou, & Monjardet, 2007; Fishburn, 1970a; Pirlot & Vincke, 1997).

1.2. Related literature

One of the most fruitful branches of modern economic theory, which has emerged from the seminal work of von Neumann and Morgenstern (1944), has been decision making under uncertainty. In many fields, such as decision theory, game theory, and financial economics, the expected utility theorem of von Neumann and Morgenstern has helped in explaining how individuals behave when they face uncertainty.

The axioms that a decision maker’s preferences have to satisfy in order for the decision maker to act as if having an expected utility function à la von Neumann–Morgenstern have been challenged by many (e.g., Allais (1953), Ellsberg (1961)). Some of these axioms are modified or removed in order to explain other types of behavior that are frequently observed in different economic settings (e.g., Gilboa and Schmeidler (1989), Kahneman and Tversky (1979)).

With a similar purpose, in this paper, we relax the transitivity axiom and try to understand and characterize the behavior of individuals, for whom indifference is not transitive, under uncertainty.

The behavior we are interested in is often discussed in various contexts when modeling bounded rationality. A decision maker may deviate from rationality by choosing an alternative which is not the optimum but that is rather “satisficing” (Simon, 1955). Similarly, a player (a decision maker in a game) may deviate slightly from rationality by playing so as to almost, but not quite, maximize utility; i.e., by playing to obtain a payoff that is within “epsilon” of the maximal payoff, as is the case for epsilon equilibrium (Aumann, 1997; Radner, 1980). What unifies such models is that the decision maker’s preferences exhibit thick indifference curves, demonstrating a weaker form of transitivity, which can be captured by intransitive indifference.

In this paper, we focus on a particular representation of semorders that provides a utility representation with a positive constant threshold as in Scott and Suppes (1958). Such representations are usually referred to as Scott–Suppes representations. Our representation theorem fully characterizes an expected Scott–Suppes utility representation that is the natural analog of the expected utility theorem of von Neumann and Morgenstern (1944).

A utility function together with a strictly positive constant threshold is said to be a Scott–Suppes representation of a semiorier if an alternative is strictly preferred to another alternative if and only if the utility of the former is strictly greater than the utility of the latter plus the (strictly positive) constant threshold. Similarly, a linear utility function together with a strictly positive constant threshold is said to be an expected Scott–Suppes utility representation of a semiorier over a set of lotteries if a lottery is strictly preferred to another lottery if and only if the expected utility of the former (with respect to the particular linear utility function) is strictly greater than the expected utility of the latter plus the particular (strictly positive) constant threshold.

The Scott–Suppes representation is initially obtained for semorders on finite sets (Scott & Suppes, 1958). Manders (1981) identifies the conditions under which semorders on countably infinite sets admit a Scott–Suppes representation. Relatively more recently, necessary and sufficient conditions for semorders on uncountable sets to have a Scott–Suppes representation have also been obtained by Cadeau and Induráin (2010). Neither of these Scott–Suppes representations focus on risky choice settings nor do they provide an expected utility representation à la Scott and Suppes (1958).

Fishburn (1968) studies semorders in the risky choice setting. He shows that if a semiorier on a set of probability distributions satisfies a particular sure-thing axiom, then indifference becomes transitive. Fishburn (1968) does not provide a representation theorem but instead points out modifications so that a preference relation representable as a semiorier might preserve intransitive indifference in a risky choice setting.

The expected Scott–Suppes utility representation, a Scott–Suppes representation in the risky choice setting, is noted as an open problem by Fishburn (1968). Two papers that focus on risky choice settings with intransitive indifference and that come close to but fall short of providing a characterization for the expected Scott–Suppes utility representation are Nakamura (1988) and Vincke (1980).

Vincke (1980) focuses on semored mixed spaces and provides a representation by obtaining a linear utility function and a non-negative threshold function. His representation provides axioms for an expected utility representation with a non-negative variable threshold. Therefore, Vincke (1980) falls short of providing axioms which would guarantee that his threshold function becomes a positive constant threshold.

On the other hand, Nakamura (1988) focuses on an interval ordered structure and provides also a representation by obtaining a linear utility function and a linear non-negative threshold function. Not only his representation provides axioms for an expected utility representation with a non-negative variable threshold but also he provides an additional axiom that gives a non-negative constant threshold. The interval ordered structures are more general structures than semored structures since every semiorier is an interval order. Yet, Nakamura’s (1988) axioms do not imply expected Scott–Suppes utility representation since his non-negative constant threshold can be zero. Furthermore, Nakamura’s (1988) representation does not provide a full characterization since as he notes a weaker axiom system than his representation might still exist. Therefore, Nakamura (1988) falls short of providing mutually independent axioms that would guarantee a positive constant threshold.

1.3. Motivation and contribution

In this paper, we provide necessary and sufficient conditions for the existence of a Scott–Suppes representation of a semiorier under uncertainty with the associated utility function being linear. Hence, our representation theorem fully characterizes the expected Scott–Suppes utility representation that is the natural analog of the von Neumann–Morgenstern expected utility theorem for semorders under uncertainty.

Our motivation for this representation includes both positive and normative perspectives. First of all, individuals seem to behave as if they cannot differentiate between probabilities that are close to each other (Kahneman & Tversky, 1979; Tversky, 1969).

In fact, with a similar observation, Allais (1953) points out the possibility to have a descriptive model of decision making under intransitive preferences. This is especially relevant when modeling economic behavior, in which it is often the case that individuals are not fully rational. Thus, the representation theorem we provide can be used to model decision making in situations where intransitivity is expected.

Fishburn (1968, p. 361); writes, referring to Scott–Suppes representation, “Its obvious counterpart in the risky choice setting is \( P \prec Q \) if and only if \( \mathbb{E}(u, P) + 1 < \mathbb{E}(u, Q) \)”, where \( P \) and \( Q \) are lotteries and \( \mathbb{E}(u, \cdot) \) is the expected utility with respect to the corresponding lottery. He finishes his paper by pointing out two routes to be explored to characterize such an expected Scott–Suppes utility representation.

This is noted in the last sentence of the conclusion section of Nakamura (1988, p. 311).
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