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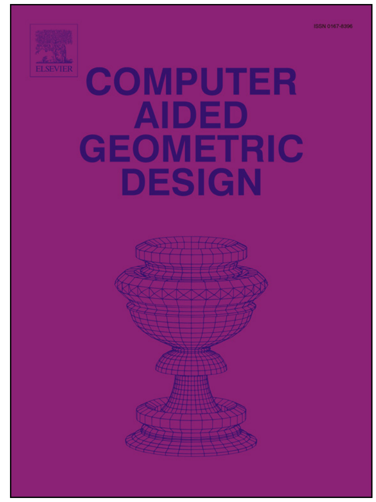

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Convergence and smoothness of tensor-product of two non-uniform linear subdivision schemes

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Abstract

The aim of this short note is to provide a rigorous proof that the tensor product of two non-uniform linear convergent subdivision schemes converges and has the same regularity as the minimal regularity of the univariate schemes. It extends results that are known for the uniform linear case and are based on symbols, a notion which is no longer available in the non-uniform setting.

Keywords: non-uniform linear subdivision scheme; tensor-product; convergence; smoothness

Definition 1 (NUSS)**.** *A univariate non-uniform (binary) subdivision scheme with* support $[0, N]$*, is given by a sequence of bi-infinite matrices* $\mathcal{A} = \{A^{[k]},\ k \geq 0\}$ *satisfying*

$$(A^{[k]})_{i,j} = 0, \quad \text{for} \quad i - 2j > N \quad \text{or} \quad i - 2j < 0, \quad i, j \in \mathbb{Z}. \tag{1}$$

One refinement step is given by

$$\mathcal{P}^{[k+1]} = A^{[k]}\mathcal{P}^{[k]}, \quad \text{for} \quad k \in \mathbb{N}, \tag{2}$$

where $P^{[k]}$ *and* $P^{[k+1]}$ *are sequences indexed by* $\mathbb{Z}$*. The subdivision scheme,* $S_{\mathcal{A}}$*, is the repeated matrix multiplication on the vector of initial data* $\mathcal{P}^{[0]} = \{P_i^{[0]},\ i \in \mathbb{Z}\}$ *by the sequence of matrices* $\mathcal{A}$*, namely*

$$\cdots A^{[k]}A^{[k-1]} \cdots A^{[0]}\mathcal{P}^{[0]}. \tag{3}$$

Example 1. *The univariate corner cutting scheme (see [1, 3, 4] for more details) is a NUSS with support* $[0, 3]$*, defined by the matrices* $\mathcal{A} = \{A^{[k]},\ k \geq 0\}$

$$(A^{[k]})_{2i,j} = \begin{cases} 1 - \gamma_i^{[k]} & \text{if} \quad j = i \\ \gamma_i^{[k]} & \text{if} \quad j = i - 1 \\ 0 & \text{otherwise} \end{cases} \qquad (A^{[k]})_{2i+1,j} = \begin{cases} 1 - \beta_i^{[k]} & \text{if} \quad j = i \\ \beta_i^{[k]} & \text{if} \quad j = i - 1 \\ 0 & \text{otherwise.} \end{cases},$$

with $0 < \gamma_i^{[k]} < \beta_i^{[k]} < 1$ *for* $i \in \mathbb{Z}$ *and* $k \geq 0$.

Extending (2) to the bivariate setting by a tensor product approach, we define,

Definition 2 (TPNUSS)**.** *Given two univariate NUSS,* $S_{\mathcal{A}}$ *and* $S_{\mathcal{B}}$ *based on* $\mathcal{A} = \{A^{[k]},\ k \geq 0\}$ *and* $\mathcal{B} = \{B^{[k]},\ k \geq 0\}$*, respectively, one refinement step of their* tensor product *NUSS, denoted by* $S_{\mathcal{A}\otimes\mathcal{B}}$*, is*

$$\mathcal{P}^{[k+1]} = A^{[k]}\mathcal{P}^{[k]}(B^{[k]})^T,$$

where $\mathcal{P}^{[k]}$ *and* $\mathcal{P}^{[k+1]}$ *are sequences indexed by* $\mathbb{Z}^2$*. The subdivision scheme,* $S_{\mathcal{A}\otimes\mathcal{B}}$*, is the repeated left and right matrix multiplication by the sequences of matrices* $\{A^{[k]},\ k \geq 0\}$, $\{(B^{[k]})^T,\ k \geq 0\}$*, respectively, acting on the initial data* $\mathcal{P}^{[0]} = \{P_i^{[0]},\ i \in \mathbb{Z}^2\}$.

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