Mechanical Systems and Signal Processing 118 (2019) 158–170

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Uncertainty quantification in the mathematical modelling of a suspension strut using BAYESIAN inference

Shashidhar Mallapur [a], Roland Platz [b,*]

[a] *Technische Universität Darmstadt, System Reliability, Adaptive Structures and Machine Acoustics SAM, Magdalenenstraße 4, 64289 Darmstadt, Germany*
[b] *Fraunhofer Institute for Structural Durability and System Reliability LBF, Bartningstraße 47, 64289 Darmstadt, Germany*

ARTICLE INFO

Article history:
Received 6 October 2017
Received in revised form 23 July 2018
Accepted 15 August 2018

Keywords:
Uncertainty quantification
Structural system
Model uncertainty
BAYESIAN inference
Posterior probability
Likelihood

ABSTRACT

In the field of structural engineering, mathematical models are utilized to predict the dynamic response of systems such as a suspension strut under different boundary and loading conditions. However, different mathematical models exist based on their governing functional relations between the model input and state output parameters. For example, the spring-damper component of a suspension strut is considered. Its mathematical model can be represented by linear, nonlinear, axiomatic or empiric relations resulting in different vibrational behaviour. The uncertainty that arises in the prediction of the dynamic response from the resulting different approaches in mathematical modelling may be quantified with BAYESIAN inference approach especially when the system is under structural risk and failure assessment. As the dynamic output of the suspension strut, the spring-damper compression and the spring-damper forces as well as the ground impact force are considered in this contribution that are taken as the criteria for uncertainty evaluation due to different functional relations of models. The system is excited by initial velocities that depend on a drop height of the suspension strut during drop tests. The suspension strut is a multi-variable system with the payload and the drop height as its varied input variables in this investigation. As a new approach, the authors present a way to adequately compare different models based on axiomatic or empiric assumptions of functional relations using the posterior probabilities of competing mathematical models. The posterior probabilities of different mathematical models are used as a metric to evaluate the model uncertainty of a suspension strut system with similar specifications as actual suspension struts in automotive or aerospace applications for decision making in early design stage. The posterior probabilities are estimated from the likelihood function, which is estimated from the cartesian vector distances between the predicted output and the experimental output.

1. Introduction

In the field of structural dynamics, mathematical models are used to predict the dynamic response of structural systems under various loading and boundary conditions. It has played a pivotal role in the research and development of engineering systems over the past decades, [1]. The stability, strength and vibrational behaviour under various operational conditions of

---

* Corresponding author.
E-mail address: roland.platz@lbf.fraunhofer.de (R. Platz).

https://doi.org/10.1016/j.ymssp.2018.08.046
0888-3270/

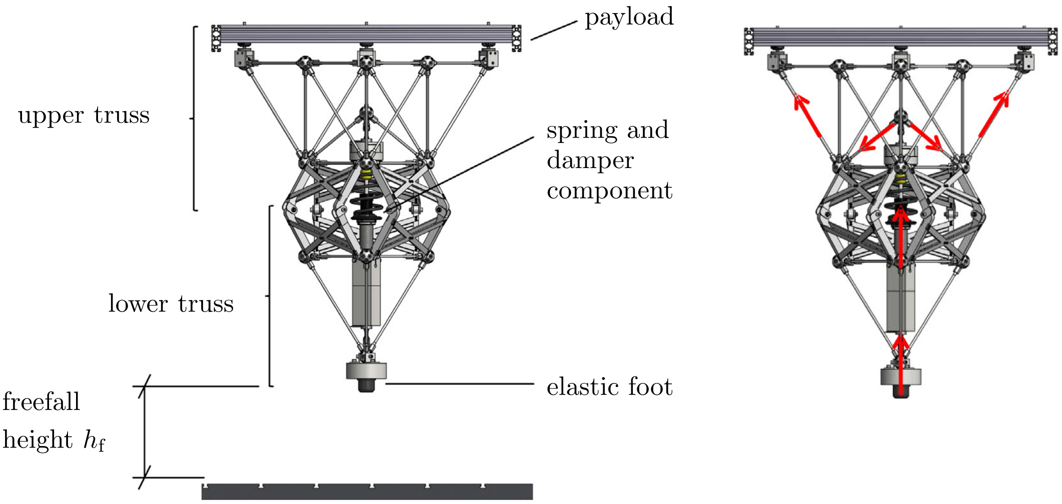
an engineering system are evaluated by the engineers using the predicted output response. However, the mathematical modelling of an engineering system can be based on different underlying assumptions about the complexity of the system, the system behaviour can be based for example on linear, nonlinear, axiomatic or empiric assumptions. Hence, there arises a need to evaluate the uncertainty in the prediction of the dynamic output by the different forms of mathematical models that describe the structural dynamic behaviour. Under the framework of verification and validation, various validation metrics have been proposed for evaluating the aleatory uncertainty in the mathematical modelling process, [2–5]. A validation metric gives a quantitative measure of the extent of agreement between the predicted model output and the experimentally observed output, [6]. These metrics are also considered for selecting an adequate model for further system development. Under the framework of BAYESIAN probabilistic approach, validation metrics that are based on posterior and prior probabilities of the output response predicted by a given mathematical model are often used to judge its agreement with the experimental data, [7]. Moreover, a validation metric can also be developed by evaluating the normalized difference between the model prediction and the experimental observations, [8]. Data uncertainty occurs when the input parameters, such as the system parameters of the mathematical models or the state variables are uncertain. This can be represented using probabilistic and non-probabilistic approaches, [9–11]. Conversely, model or model form uncertainty occurs when the functional relations between the model input and the model output as parameters and state variables are uncertain.

As a new approach, the authors present a way to adequately compare different models based on linear, nonlinear, axiomatic or empiric assumptions of functional relations. In this contribution, the authors consider the posterior probability of mathematical models as a model comparison metric to quantify the uncertainty in the mathematical modelling of a suspension strut, [12]. The posterior probability is estimated from the likelihood function, which is in turn proportional to the cartesian vector distances between the numerically simulated response and the experimental response, [13]. This work considers a suspension strut which is referred as the modular active spring-damper system (German acronym MAFDS). The MAFDS is developed under the framework of the German Collaborative Research Center SFB 805 at the Technische Universität Darmstadt. The concept of the MAFDS is registered as a Patent DE 10 2014 106 858.A1 “Load transferring device”, [14].

2. Modular Active Spring Damper System MAFDS

Fig. 1 illustrates the MAFDS. The MAFDS carries a payload on an upper truss that is connected to the lower truss via guidance links. The guidance link enables the kinematic motion between the lower and upper truss structures. The elastic foot acts as an impact absorption element which introduces the axial and lateral forces due to impact into the MAFDS. The axial forces are transferred to the upper truss via the spring-damper component, Fig. 1b. An impact scenario is created by releasing the MAFDS from a specified drop height *h*. Uncertainty is investigated especially for MAFDS's main operating function and purpose: sustain stability, well balanced load distribution and ability to attenuate vibrations.

The MAFDS serves as an academic demonstrator to investigate the different methods and technologies to control uncertainty in all development stages such as the conceptual phase, design and optimization phase, production and assembly phase as well as the final operational phase. Passive, semi-active and active technology approaches for stability, load distribution and vibration control using sensor and controlled actuator networks are developed in the SFB 805 to compensate uncertainty in the main functions mentioned above, [16–22]. In addition, solutions for active damage control such as lowering the growth rate of fatigue cracks with piezoelectric actuators inducing forces near the crack tip like in [23] are considered. The MAFDS has similar dynamic requirements of a typical suspension strut such as an aircraft landing gear. However it



(a) (b)

Fig. 1. (a) CAD illustration of suspension strut MAFDS, (b) Load distribution in the MAFDS.