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# Condensation of the correlation functions in modal testing

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A B S T R A C T

A well known issue when performing Operational modal analysis in the time domain is the challenge of choosing the right order of the system matrix. Setting the order higher than the rank of the system results in over fitting, i.e. having more poles to place than modes of the system. This results in fitting poles to noise or non linearities of the system. In This paper a method to avoid over fitting by reducing the physical channels to a reduced number of pseudo channels condensing the relevant physical information and leveling the level of energy in each channel is suggested.



## 1. Introduction

Modal testing can be regarded as the process of condensing information about the physical properties of a structure from experimental measured data into a set of modal parameters such as natural frequencies, damping ratios and mode shapes. When performing modal analysis there are many challenges and user required settings, which makes this a highly specialized task. The sensor layout and selection of appropriate sensors is of high importance. Often a large amount of sensors are used to achieve a proper description of the physical behavior of the structure. In order to extract information from the measured signal a mathematical model is fitted either in the time or in the frequency domain. A large number of channels gives a high resolution, but also results in comprehensive computational effort to fit the mathematical models. This is especially an issue when using time domain techniques to estimate the modal parameters. Therefore a condensation of the number of Degrees Of Freedoms (DOF) can be very beneficial and in some cases where the physical nature of the structure or the excitation is challenging it can improve the estimation of the modal parameters. The classical time domain techniques such as the Ibrahim Time Domain (ITD) [1], the Time Domain Polyreference (TDPR) [2] or the Eigen Realization Algorithm (ERA) [3] are all based on a least square fit to free decays [4], e.g. within Operational Modal Analysis(OMA) the correlation functions [5].

When fitting in the time domain all modes are presents in the time signal. This means that the number of poles is equal to the number of modes hereby leaving no poles to fit noise or non-linearities. This leads to under fitting where some modes are missed or poorly estimated [4]. Band pass filtering the time signal into frequency bands containing a limited set of modes can meet this problem. In cases with many DOF's over fitting can occur, when having to many poles to fit to a limited set of physical modes. Then the contribution from noise and non-linearities will have a significant influence and it will be difficult to

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distinguish spurious poles from physical ones. This leads to the need to condensate the information in the time signal into a reduced set of DOF's [6].

A well known approach to condense information is Principal Component Analysis (PCA), which has been widely used within data analysis and signal processing. The idea behind PCA is to project a set of possibly correlated data into a space of linearly uncorrelated components. Where the variance of the new components successive define each component by assuming orthogonality. The first component represent the largest variation of the data set [7]. This transformation is often performed using the Singular Value Decomposition (SVD) [8]. More recently the introduction and rapid development of the machine learning environment has introduced evolutions of the PCA to perform identification of the modal parameters such as the Independent Component Analysis (ICA) [9,10]. ICA builds on the principles of PCA and aims to recover the source signal as a combination of instantaneous linear mixtures [11]. The principal of PCA has widely been used in modal testing since early 1970's, where the computational and software capacities where developed[12]. Some of the most well known approaches of PCA within modal testing is the Complex Modal Indicator Function (CMIF) [13] within Experimental Modal Analysis (EMA), and the Frequency Domain Decomposition (FDD) within OMA [14,15].

The time domain identification technique ERA uses the principles of PCA to reduce the size of the Hankel matrix in order to reduce the rank of the system hereby sorting the physical poles from the spurious poles of the system. This is done by decomposing the first column in the Hankel matrix by the use of the singular value decomposition. In the ideal case the non-zero singular values will contain the information of the system and the number of non-zero singular values reveal the rank of the system. Hereby constructing a reduced system from which the modal parameters are estimated.

In this paper condensation of DOF's using time domain identification techniques is formulated in a general formulation without being depended on the identification technique. The approach can be used within the scope of both OMA and EMA as the impulse response functions and the correlation functions share the same properties [5]. The idea is to reduce the physical channels to a new set of pseudo channels containing the relevant physical information and normalizing the level of energy in each channel. Using the first time lag of the correlation functions as this contains information of all modes present in the signal. By applying condensation directly to the correlation functions the amount of data to process is reduced and opens for a free choice of identification technique to be used to estimate the modal parameters. The method presented in this paper should be considered as a tool to enhance and isolate the physical poles and to remove noise poles from the estimated correlation functions. In order to apply the method, the user needs to define a suitable number of modes to estimate, hereby defining the number of pseudo DOF's to use. In order to assess the number of modes in the measured signal, different techniques can be used, e.g. the FDD-plot, as described in [4].

The outline of the paper follows with the theory behind condensation which is introduced in the second section. It is shown how the SVD is used to condense and transform the full signal into a reduced set of pseudo channels where the normalization of the data captures the energy of the signal in a set of pseudo DOF's that are uncorrelated and can be thought of as an ideal set of sensors that contains the same information. The third section illustrate the use of condensation by a simulation case of a 20 DOF system, where each step in condensation is illustrated and the effect on the error of modal parameters is shown using monte carlo simulation. In the fourth section condensation is applied to a data set from the cabled stay bridge Port Mann Bridge in order to illustrate condensation of data from a real structure. In the end of the paper the properties of condensation is discussed.

## 2. Theory

System identification in the time domain within Operational Modal Analysis is based on the correlation function [5] defined by

$$\mathbf{R}(\tau) = \mathbf{E}[\mathbf{y}(t)\mathbf{y}^T(t+\tau)] \tag{1}$$

where $\mathbf{y}$ is the measured response as column vector, $\tau$ is the time delay, and $\mathbf{E}[\cdot]$ is the expectation value operator. For dynamic responses that are assumed ergodic and stationary, it can be shown that this can be written as [4]:

$$\mathbf{R}(\tau) = \sum_{n=1}^{2N} \gamma_n \mathbf{b}_n^T e^{\lambda_n \tau} \tag{2}$$

where $\mathbf{b}_n$ is a mode shape, $\gamma_n$ is the participation vector for the mode and $\lambda_n = -\zeta\omega_{0n} + j\omega_{0n}\sqrt{1-\zeta_n^2}$ is the pole of the mode. The pole contains the angular frequency, $\omega_{0n}$ and the damping ratio $\zeta_n$. The system response is assumed to be described using $2N$ poles, giving $N$ mode shapes.

For zero time delay $\tau = 0$ Eq. (2) shows that the correlation matrix $\mathbf{R}_0 = \mathbf{R}(0) = \mathbf{E}[\mathbf{y}(t)\mathbf{y}^T(t+\tau)]$ can be written

$$\mathbf{R}_0 = \sum_{n=1}^{2N} \gamma_n \mathbf{b}_n^T \tag{3}$$

The correlation describes the level of correlation between the coordinates in the vector $\mathbf{y}$. The diagonal of the correlation matrix contains the variance of the coordinates of $\mathbf{y}$.