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An analytical investigation into the propagation properties of uncertainty in a two-stage fast Bayesian spectral density approach for ambient modal analysis

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ABSTRACT

This paper investigates the uncertainty propagation properties of a two-stage fast Bayesian spectral density approach (two-stage fast BSDA) with separated modes proposed previously by the authors Yan and Katafygiotis (2015a,b) [1,2], deriving explicit formulas for the dependence of the posterior coefficients of variation (c.o.v.) of the identified modal parameters in terms of different parameters influencing the identification. For this, an approximation analysis strategy proposed in Au (2014a,b) [3,4] is adopted. Although the explicit closed-form approximation expressions are relatively complex, the expressions for the approximate dependence of uncertainty are simple and informative. The analysis reveals a strong correlation among the prediction error, the damping ratio and the power spectral density (PSD) of the modal excitation. While similar correlation trends have been observed in the posterior uncertainty analysis of the fast Bayesian FFT (fast BFFT) approach Au (2014a) [3], the present method shows that the identification results are more sensitive to modeling error. Note that this is not a contradicting result, as the uncertainty propagation properties of different methods may generally differ. Note that fast BFFT is a more fundamental method, in the sense that it processes FFT data directly, while the two-stage fast BSDA uses spectral density data in a manner that allows for decoupling of the mode shape data. Validation studies using synthetic data and field data measured from a laboratory model provide a practical verification of the rationality and accuracy of the theoretical findings.

1. Introduction

Operational modal analysis (OMA) has gained increasing popularity in both theoretical developments and practical applications [5–14]. Compared to forced vibration testing or free vibration testing, ambient vibration testing is attractive for civil engineering structures [13]. At the same time, ambient vibration testing faces challenges related to the difficulty to control the testing environment, and the need to model the unmeasured loading by a stochastic process [3]. Therefore, uncertainty plays a significant role in OMA [13], suggesting that it ought to be tackled as a statistical inference problem, with a need to ensure the robustness of the identification results.

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A number of statistical approaches have been developed to quantify the uncertainty of OMA over the past decades. They are generally divided into two main broad approaches, e.g., the non-Bayesian approaches and Bayesian approaches [13]. Prominent references of non-Bayesian approaches include the development of frequency-domain maximum likelihood (ML) techniques [15–22] and stochastic subspace identification (SSI) based methods [23–28]. ML identification as an optimization-based method was originally formulated in [15–17]. The ML technique results in nonlinear equations of the unknown parameters. To address the drawbacks of ML estimators due to the difficulties of being computer intensive and handling large amounts of data, the original algorithm has been modified using adapted parameterization and fast signal processing technique [18]. A few years later, the ML method was also extended from frequency response functions to power spectra as primary data in output-only cases [19]. More recently, new maximum likelihood modal model-based (ML-MM) modal identification methods have been proposed [20–22]. The modal parameters in ML-MM methods are identified by directly fitting the modal model to the measured frequency-domain data (e.g., FRFs or power spectra) in an ML sense. The cost function to be optimized is the sum of the squared absolute values of the weighted errors between the model and the measurements. It has been pointed out that the proposed approaches are more numerically stable than the conventional rational fraction polynomial model.

SSI-based covariance estimates on the identified parameters have also been studied extensively [23–28]. A detailed formulation of the covariance computation for the modal parameters based on the propagation of first-order perturbations from the data to the identified parameters was given in [23–25]. The covariance computation for multi-setup subspace identification was investigated in [26]. A fast computation scheme was proposed in [27] to reduce the computational burden of the uncertainty computation scheme of SSI. More recently, novel variance computation schemes for modal parameters were developed for four subspace algorithms including output-only and input/output methods, as well as data-driven and covariance-driven methods [28].

Substantial development has also been achieved in structural dynamics using the Bayesian statistical framework [29–37]. In the field of OMA, Bayesian approaches in the time domain [38,39] and in the frequency domain using the spectral density [40] and Fast Fourier Transform (FFT) [41] have been proposed by Yuen and Katafygiotis. Yuen published the first monograph on this topic [42]. These works lay a mathematically rigorous theoretical foundation for statistical operational modal properties although, unfortunately, the original formulations are relatively computationally demanding. To address the computational challenges of the conventional BFFT approach [41], a novel contribution was made by Au [30,43,44] through employing linear algebra techniques. Based on the formulation for FFT data, fast algorithms amenable to mathematical analysis have been developed for extracting the most probable values as well as their posterior covariance matrix. An overview of this approach, including issues of theoretical, computational and practical nature was available in [45]. A more comprehensive introduction to this topic covering theoretical formulations, computational algorithms, and practical applications has also been presented in a new monograph [13].

Following the work of fast BFFT approach [30,43,44] and Bayesian spectral density approach (BSDA) [40], a two-stage fast BSDA was formulated more recently by the authors [1,2]. A variable separation technique was proposed to decouple the interaction between the spectrum variables (e.g., frequency, damping ratio as well as PSD of modal excitation and prediction error) and the spatial variables (e.g., mode shape components). The sum of auto-spectral density can be processed by 'fast Bayesian spectral trace approach' (FBSTA) in the first stage for making statistical inference on the the spectrum variables, while the spatial variables (mode shapes) can be identified through 'fast Bayesian spectral density approach' (FBSDA) in the second stage by processing the information contained in the entire power spectral density matrix. Due to the use of variable separation technique, the dimension involved in computation is reduced. It is easily-implemented without resorting to calculating the cross spectral density matrix in the first stage. Furthermore, it is able to incorporate information contained in all measured dofs corresponding to different setups [1,2]. The proposed two-stage fast BSDA has been successfully applied to a long-span suspension bridge [46].

Mathematically speaking, the two-stage fast BSDA share some commons features with the ML technique [15–22] when no prior information is incorporated. For example, each of them should formulate a negative log-likelihood function in the frequency domain and optimize this cost function to obtain the modal properties. However, they still differ in several aspects: (i) In non-Bayesian statistics, the definition of probability is a long-run limiting relative frequency. In Bayesian statistics, however, probability is viewed as a measure of the plausibility (a personal degree of belief in a proposition) of a proposition conditional on information. In Bayesian OMA, the modal parameters included in a modal class are viewed as uncertain parameters or random variables. (ii) In Bayesian OMA $s_\mu \boldsymbol{I}_{n_o}$) approaches, the covariance of prediction error between the model and measurements (i.e., are viewed as unknown parameters to be identified. In the ML-MM method, the covariance of the equation errors is estimated prior to modal identification using techniques such as the estimator of the nonparametric identification of the FRFs [20,21]. Therefore, in the Bayesian OMA context, the parameters to be identified include the natural frequencies, damping ratios, mode shapes, PSD of the prediction error, and modal excitation [1,2]. The parameters in ML-MM techniques include the poles, participation factors, mode shapes, and lower and upper residual terms [20,21]. The Bayesian OMA makes inferences about the modal parameters as well as the PSD of the prediction error and modal excitation by processing the information contained in raw measurements to obtain the uncertainty of the parameters to be identified. (iii) The ML technique can take advantage of the quadratic form of the cost function. In real applications, the objective function of BSDA is not quadratic form, which is more computationally demanding as illustrated in [1,2]. In this regard, the BSDA is more restrictive on the assumption that the contribution of response from other modes are ignored.