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A multivariate interval approach for inverse uncertainty quantification with limited experimental data

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ABSTRACT

This paper introduces an improved version of a novel inverse approach for the quantification of multivariate interval uncertainty for high dimensional models under scarce data availability. Furthermore, a conceptual and practical comparison of the method with the well-established probabilistic framework of Bayesian model updating via Transitional Markov Chain Monte Carlo is presented in the context of the DLR-AIRMOD test structure. First, it is shown that the proposed improvements of the inverse method alleviate the curse of dimensionality of the method with a factor up to 10^5. Furthermore, the comparison with the Bayesian results revealed that the selection ofthe most appropriate method depends largely on the desired information and availability of data. In case large amounts of data are available, and/or the analyst desires full (joint)-probabilistic descriptors of the model parameter uncertainty, the Bayesian method is shown to be the most performing. On the other hand however, when such descriptors are not needed (e.g., for worst-case analysis), and only scarce data are available, the interval method is shown to deliver more objective and robust bounds on the uncertain parameters. Finally, also suggestions to aid the analyst in selecting the most appropriate method for inverse uncertainty quantification are given.

1. Introduction

In general engineering practice, the knowledge on a structure is usually incomplete, be it due to inherent variable model parameters or a lack of knowledge on the true parameter values [1,2]). Hence, representing these model parameters as deterministic quantities might prove to be inadequate when a reliable and economic design is pursued, as a large degree of conservatism is needed to prevent premature failure and corresponding maintenance or insurance costs. This over-conservatism not only impairs the economic cost of producing the component; it also leads to unnecessary weight increase, which is impermissible in high-performance sectors such as machinery design, aerospace or automotive. In the last few decades,

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highly advanced techniques including probabilistic [3], possibilistic [4] or imprecise probabilistic methods [5] have been introduced to include non-determinism efficiently in these design models.

In order for these tools to deliver a realistic quantification of the non-determinism in the responses of the design model, the description of the non-deterministic parameters of the model should be made objectively and accurately. Since not all parameters (such as e.g. connection stiffness values or heterogeneous material properties) are trivial to measure directly, inverse uncertainty quantification (UQ) techniques have been introduced. Following inverse UQ, the responses of the structure are measured and used to infer knowledge on the non-determinism in the model parameters. As concerns inverse UQ in a probabilistic sense, the class of Bayesian methods is considered the standard approach [6,7], even for random fields [8,9]. However, in the context of limited, insufficient, vague or ambiguous data, the prior estimation of the joint probability density function of the non-deterministic parameter values is subjective. Moreover this estimate influences the quantified result to a large extent when insufficient independent measurement data are available.

Inverse UQ methods for the identification and quantification of multivariate interval uncertainty usually minimise a squared $\mathcal{L}_2$-norm over the difference between the interval boundaries of respectively a measurement data set and the prediction of the FE model [10,11]. Application of most of these techniques is prone to ill-conditioning and non-uniqueness when no special care is taken in the definition of the identification problem [12]. Also alternative approaches using Kriging predictor models were introduced recently [13,14].

Recently, a novel methodology for the identification of multivariate interval uncertainty was introduced by some of the authors in [15,16], with an extension to interval fields in [17,18]. This method is based on the convex hull concept for the representation of dependent uncertain output quantities of an interval FE model, and iteratively minimises the discrepancy between the convex hull of these uncertain output quantities with the convex hull over a set of replicated measurement data. However, since the computation of a convex hull follows an exponential time complexity with its dimension, the dimension of these convex hulls should be reduced as to allow for applying this method to large-scale problems. Dimension reduction is a topic that is quickly emerging in the fields of big data and machine learning, where datasets are often too high-dimensional to be handled directly. In this context, a broad range of techniques based on for instance covariance matrix decompositions [19], manifold learning approaches [20], or active subspace methods [21] have been introduced in recent years. In the context of the inverse quantification of multi-dimensional interval uncertainty, the application of such dimension reduction methods is an under-explored domain.

Finally, whereas the literature on comparing forward UQ in a probabilistic and non-probabilistic context is abundant [22–24], such a practical comparison for inverse approaches is severely lacking in literature. The objective of this paper is therefore twofold. First, an improved version of a recently proposed interval method [16,17] is presented in the sense that by reducing the dimension of the corresponding convex hulls, more challenging problems can be tackled. In addition, an objective comparison of Bayesian uncertainty quantification methods (see e.g., [6,7,25] or Section 3), which are most commonly applied in a probabilistic context is provided and suggestions for choosing the most appropriate technique based on the data are made.

The paper is structured as follows. Section 2 introduces the extensions to the novel method for the identification and quantification of multivariate interval uncertainty. It is illustrated that the exponential time complexity of computing the objective function is relaxed by projecting the convex hull onto lower-dimensional subspaces of an orthogonal basis with a dimension equal to the effective dimension of the convex hulls. Section 3 presents the reader with the concept of Bayesian uncertainty quantification. Both techniques are critically compared and a conceptual comparison is given in Section 4. Section 5 presents a case study comparing the applicability of both methods to the well-known DLR-AIRMOD [14] case. Specifically, it is studied how both methods perform in terms of obtained information, computational cost and accuracy, depending on the size of the dataset. Finally, Section 6 lists the conclusions of this work.

2. Multivariate interval quantification

This section introduces the interval finite element method and the method used for the identification and quantification of multivariate interval uncertainty based on indirect measurement data. In the following, a model parameter θ having interval uncertainty is denoted θ^I. Vectors are expressed as lower-case boldface characters $\boldsymbol{\theta}$. Interval parameters are either represented using the bounds of the interval $\theta^I = [\underline{\theta}; \overline{\theta}]$ or the centre point $\hat{\theta} = \frac{\underline{\theta}+\overline{\theta}}{2}$and the interval radius $r_x = \frac{\overline{\theta}-\underline{\theta}}{2}$.

2.1. The interval finite element method

Let $\mathcal{M}$ be a deterministic Finite Element model that is used to solve a (set of) differential equations for $\boldsymbol{z^m} \in \mathbb{R}^d$ through the vector valued function operator g:

$$\mathcal{M}(\boldsymbol{\theta}) : \boldsymbol{z^m} = g(\boldsymbol{\theta}), \qquad g : \mathbb{R}^k \mapsto \mathbb{R}^d \tag{1}$$

with $\boldsymbol{\theta} \in \mathcal{F} \subset \mathbb{R}^k$ the vector of model parameters and $\mathcal{F}$ the sub-domain of feasible parameters (e.g., non-negative contact stiffness).