Mechanical Systems and Signal Processing 118 (2019) 549–567

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

On a space-time regularization for force reconstruction problems

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ARTICLE INFO

Article history:
Received 8 March 2018
Received in revised form 8 August 2018
Accepted 2 September 2018

Keywords:
Inverse problem
Force reconstruction
Space-time approach
Multiplicative regularization
Generalized-α method

ABSTRACT

Tikhonov and LASSO regularizations are commonly used to solve force reconstruction problems in time domain. Unfortunately, these particular forms of additive regularization are not well adapted to tackle both localization and time reconstruction problems simultaneously, since they are generally restricted to the reconstruction of sources sharing the same space and time characteristics. To alleviate this limitation, a multiplicative space-time regularization is introduced. The proposed regularization strategy takes advantage of one's prior knowledge of the space-time characteristics of excitation sources. It also introduces a novel reconstruction model based on the generalized-α method, which is unconditionally stable and second-order accurate. The validity of the proposed method is assessed numerically and experimentally. In particular, comparisons with standard regularization terms point out the practical benefit in exploiting both spatial and temporal prior information simultaneously in terms of quality and robustness of reconstructed solutions.

1. Introduction

The resolution of force reconstruction problems in time domain still remains an active topic in the mechanical engineering community as suggested by the intensive dedicated literature. Except particular strategies [1–8], the most widespread approaches to solve this problem nowadays are certainly Kalman filtering [9–12] and regularization techniques [13–16]. However, both methods operate in different context. Kalman filtering is an online strategy making assumptions on the evolution of the systems [17,18]. On the contrary, regularization is an off-line approach that allows exploiting one's prior knowledge on the sources to identify through the definition of the regularization term [16,19–21]. The latter characteristic is at core of the present paper and explains why the next of this introduction is only focused on the analysis of existing regularization strategies.

In general, two categories of reconstruction problem can arise in practical situations. The first one is related to the localization of excitation sources, while the second one consists in reconstructing the time signal of prelocalized sources. Regarding the localization problem, the regularization term reflects the spatial prior information available on the sources to reconstruct. It is often expressed as the ℓ_q-norm of the solution sought. Such a norm is flexible enough to express one's prior knowledge on the forces to identify, since smooth solutions are obtained for $q = 2$ [22,23], while localized excitation fields are promoted for $q \leqslant 1$ [24–26]. On the other hand, for reconstructing the excitation signal of prelocalized sources, the

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https://doi.org/10.1016/j.ymssp.2018.09.002

regularization term has to reflect the associated prior information. It should be noted that the choice of a particular value of q is strongly dependent on the length on the time window on which the reconstruction is performed. Indeed, a signal can be considered continuous over a short duration and localized on the long term. Generally, when the force signal exhibits a certain continuity, the regularization term is defined from the ℓ_2-norm of the solution vector to identify and leads to the Tikhonov regularization [13,14,27]. On the contrary, when the excitation signal is rather impulsive, the regularization term is constructed from the ℓ_1-norm and gives rise to the LASSO regularization [16,28]. Consequently, it appears that most of the methods proposed in the literature are theoretically not always well adapted to tackle both the localization and time reconstruction problems at the same time, except for configurations where the force vector to identify has the desired structure or the spatial distribution of the sources and the nature of the excitation signals share the same space-time characteristics[1] such as the sparsity as shown in Refs. [16,29,30]. To the best of our knowledge, only a few methods have been developed to address these issues [31–34]. However, these methods generally addressed the space-time reconstruction problem in a separated manner [35–38].

It is thus of primary interest to simultaneously exploit both the spatial and the temporal features of excitation sources to constrain the space of admissible solutions to aid the reconstruction process in finding an optimal solution. These requirements are actually met by regularization terms derived from mixed $\ell_{p,q}$-norms. Indeed, mixed $\ell_{2,q}$-norm for $q \leqslant 1$ has revealed all its potential in signal and image recovery applications [39–41]. In the context of force reconstruction, Rezayat et al. first introduced an additive regularization using a regularization term based on a mixed $\ell_{2,1}$-norm to identify broadband point forces in the frequency domain [42]. Recently, Aucejo and De Smet have extended this idea by developing, for frequency domain applications, a multiplicative regularization based on the definition of a mixed $\ell_{p,q}$-norm to accurately reflect experimenters knowledge on the type (localized or distributed) of the excitation forces, as well as on the nature of the force spectrum [20].

In the present paper, a space-time regularization is proposed for time domain applications in order to solve both localization and time reconstruction problems within a unique framework. Actually, the proposed method relies on three pillars. The first one is related to the definition of the reconstruction model. Here, the reconstruction model is obtained from a state-space model of the structure built from a generalized-α integration scheme [43]. In doing so, the reconstruction model is unconditionally stable and second-order accurate. The second key feature is the space-time regularization term defined, as in [20], from a mixed $\ell_{p,q}$-norm to properly exploit the space-time information available on the sources to identify. The last pillar of the proposed approach concerns the formulation of the inverse problem, since it is based on the multiplicative regularization recently introduced by the authors in the context of mechanical source identification [20,44,45]. This particular strategy is generally computationally more efficient than the corresponding additive regularization, because it is free from the preliminary definition of any regularization parameter [44]. This explains its use in the present paper. To clearly introduce the main features of the proposed regularization strategy, this article is divided into four parts. Before considering the core of the paper, the need for another regularization strategy for dealing with time domain applications is explained in Section 2. Section 3 is devoted to the introduction of the three pillars of the space-time regularization, as well as that of the related resolution algorithm. Numerical and experimental validations of the space-time regularization are proposed in Section 4 and 5. Obtained results point out the practical interest in exploiting both spatial and temporal prior information simultaneously in terms of quality and robustness of reconstructed solutions.

2. The need for another regularization method in time domain

As highlighted in the introduction, the most widespread regularization strategies to solve the source identification problem in time domain are the Tikhonov regularization [13–15,46,47] and the LASSO regularization [16,29,30,28], which belong to the class of additive regularization methods. The related multiplicative regularizations can be formally written under the following generic form:

$$\widehat{\mathbf{F}} = \underset{\mathbf{F}\setminus\{\mathbf{0}\}}{\operatorname{argmin}} \, \|\mathbf{Y} - \mathbf{H}\mathbf{F}\|_2^2 \cdot \|\mathbf{F}\|_q^q, \tag{1}$$

where $\mathbf{H}$ is the convolution matrix, $\mathbf{Y}$ is the measured output vector, $\mathbf{F}$ is the force vector and q is the norm parameter. This formulation gives rise to the multiplicative Tikhonov regularization (mTIK) when $q = 2$ and to the multiplicative LASSO regularization (mLASSO) when $q = 1$.

Unfortunately, these formulations can lead to inaccurate reconstructions when one wants to identify both the location and the time signal of the excitation field. Indeed, the norm parameter q helps to exploit one's prior knowledge on the sources to identify, since choosing $q = 2$ promotes distributed solutions [23,26], while setting $q \leqslant 1$ enforces the sparsity of the solution vector [48,49]. In other words, mTIK and mLASSO regularizations are generally limited to the reconstruction of sources sharing the same space and time characteristics such as the sparsity (e.g. reconstruction of a hammer impact).

To illustrate this, let us consider the reconstruction of a harmonic point force imposed by a shaker on the simply-supported 1 m-long beam described in Section 4 at $x_0 = 0.3$ m from its left end [see Section 4 for details]. To implement

[1] This is typically the case of a hammer impact excitation reconstructed over a long duration, for which the corresponding spatial distribution is sparse (point force) as well as the time excitation signal (impulsive excitation). Another example is a distributed harmonic excitation.