Signal Processing 154 (2019) 159–167

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

H_∞ state estimation via asynchronous filtering for descriptor Markov jump systems with packet losses

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A R T I C L E I N F O

Article history:
Received 23 February 2018
Revised 11 August 2018
Accepted 2 September 2018
Available online 3 September 2018

Keywords:
Descriptor Markov jump systems
Packet losses
Asynchronous filter
H_∞ state estimation

A B S T R A C T

This paper is concerned with the problem of asynchronous state estimation for a class of discrete-time descriptor Markov jump systems with packet losses. Packet losses and asynchronous modes are simultaneously considered in the communication link between the plant and the estimator, where the missing probability of packet losses is governed by a binary distribution. Firstly, by constructing a stochastic Lyapunov functional, a sufficient condition is given such that the filtering error system is stochastically admissible and has a prescribed H_∞ noise attenuation performance index. Then, based on the matrix inequality decoupling technique, a linear matrix inequality (LMI)-based condition on the existence of the H_∞ asynchronous filter is presented, where the asynchronous filter can be of full-order or reduced-order. Compared with the previous methods, the proposed design method does not impose constraints on slack variables and dimensions of the designed filter, which shows less conservative results. Finally, numerical examples are given to illustrate the benefit and applicability of the new method.

1. Introduction

Markov jump systems (MJSs) have received increased attention in the process industry in recent years, which attribute to its effective ability of describing the abrupt failures or sudden environmental changes in practical systems. MJSs have found wide applications in such areas as networked control systems, economic systems, biological systems, power systems and so on [1]. As a consequence, lots of elegant results have been presented for MJSs. To just mention a few, the problems of stability analysis and controller design for MJSs were studied in [2–4]. The authors in [5–8] discussed the issues of the filtering for MJSs, where the designed filters are mode-dependent or mode-independent. However, the above literatures cannot cope with the complex asynchronous phenomenon about modes information between the system and the filter. To deal with this challenge, the asynchronous control and filtering for MJSs were studied in [9,10]. It should be noted that all the mentioned literatures are concerned with the regular MJSs.

Descriptor systems are also called singular systems that can effectively describe a larger number of practical systems, such as biological systems, network control systems, economic systems, power systems and so on [11]. Descriptor systems are much more complicated than regular systems because the solution of descriptor systems may contain infinite modes. Thus, for descriptor systems, not only the stability need to be considered, but also the regularity (impulsive-freeness for continuous-time systems) need to be considered, which guarantees the existence and uniqueness of the solution for the systems. Note that random failures or repairs of components, sudden environmental disturbances, changing subsystem interconnections, abrupt variations may occur in the descriptor plants. We naturally model them as descriptor MJSs (DMJSs) [12]. Such models are commonly found in practical engineering systems, many analysis results and design theories for DMJSs have been developed. For the practical areas, the applications on the electrical circuit in [13], the inverted pendulum devices in [14] and the oil catalytic cracking process in [15] were concerned. On its theoretical sides, there are also many elegant results. For example, by applying two equivalent sets, necessary and sufficient conditions of dissipative control for singular MJSs were proposed in [16]. An observer-based sliding-mode controller design for singular MJSs has been synthesized in [17]. By imposing hard restrictions

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https://doi.org/10.1016/j.sigpro.2018.09.003

on the key freedom matrices M and N, the issue of the $l_2 - l_\infty$ filter design of DMJSs was designed in [18]. By introducing some matrix inequalities, the problem of the filter design for singular MJSs was given in [19,20]. Authors in [21] considered the extended passive filtering problem for singular MJSs with time-varying delays, wherein the hard restriction on the key freedom matrices G_i and the matrix inequality $-G_iX_i^{-1}G_i^T \le X_i - G_i^T - G_i$ were used. The restrictions on freedom matrices and the matrix inequalities are used in [18–21], which gave conservative results. How to reduce the conservatism caused by these restrictions is one of the directions in this paper. Note that in the above mentioned results, the communication link has been assumed to be perfect when exchanging data among devices connected to the shared medium. However, in practice, the data may be lost while in transit through the networks due to limited bandwidth.

Networked control systems (NCSs) have many advantages and fruitful applications in a broad range of areas such as automated highway systems, remote surgery and mobile sensor networks. Different from the traditional control systems, the data may be lost in the network. Over the past decades, due to their significance both in theory and application, there has been increasing interest in the control synthesis and estimation problems for NCSs. Note that the effects of packet losses can degrade the system performances or even cause faults, many useful results on designing NCSs against the packet losses have been investigated [22–32]. For example, by viewing the packet loss as a binary switching sequence that obey the Bernoulli random binary distribution, the design of controllers for NCSs has been concerned in [28–30]. For NCSs with MJSs plants, the finite-time energy-to-peak filtering problem has been studied in [32]. However, the aforementioned results are concerned with state-space plants (i.e. regular plants). For NCSs with singular plants, the H_∞ filtering problem was investigated in [33,34], and the fault detection issue was discussed in [35], respectively. By setting $G_{4i}^{-1}G_{3i} = I$ in free-weighting matrix G_i, the problem of fault detection filter design for discrete-time singular MJSs with intermittent measurements has been proposed in [36], wherein the dimension of the filter is actually of full-order. By defining $X_i = \begin{bmatrix} X_{1i} & Y_i \\ X_{2i} & Y_i \end{bmatrix}$ and using the matrix inequality $-X_i\hat{P}_i^{-1}X_i^T \le \hat{P}_i - X_i^T - X_i$, the issue of the non-fragile filter design was addressed for discrete-time singular MJSs subject to missing measurements in [37]. By setting $J_{im} = \begin{bmatrix} J_{1im} & b_1X \\ J_{2im} & b_2X \end{bmatrix}$, the energy-to-peak filter design for networked singular MJSs over a finite-time interval was investigated in [38]. It should be pointed out that the result in [36–38] is not only with some restrictions on free-weighting matrix but also limits the dimension of the filter to be of full-order. A natural research problem is how to establish a less conservative filter design condition for networked DMJSs without any constraint on the free-weighting matrix and the dimension of the filter.

In this paper, we investigate the asynchronous state estimation problem for discrete-time DMJSs with packet losses. By taking the packet losses and asynchronous modes into consideration, a novel filtering error system has been proposed. Firstly, based on a stochastic Lyapunov functional, a sufficient condition is obtained such that the filtering error system is stochastically admissible with the H_∞ noise attenuation performance index. Then by using the matrix inequality decoupling technique, the condition on the existence of the asynchronous filter is developed in terms of strict LMIs. The main novelty of the paper lies in the following aspects: 1) By using the compensation scheme of packet losses, the asynchronous state estimation problem for discrete-time DMJSs affected by packet losses is studied for the first time. 2) Compared with [18,21,36–38], the filter design method in the paper shows less conservative results since the constraints on the free-weighting matrix and the dimension of the filter are overcome. Last, two numerical examples are provided to show the effectiveness of the proposed methods.

Notations: Throughout this paper, the following standard notations are used (Table 1).

Table 1
The notations and descriptions.

Notation	Description
$X \ge 0$ $(X > 0)$	Semi-positive definite (positive definite) matrix
I	Identity matrix with appropriate dimensions
0	Zero matrix with appropriate dimensions
$(\cdot)^T$	Transpose of a matrix
diag$\{\cdots\}$	Block-diagonal matrix
$\|x\|$	Euclidean norm of the vector x
$\mathbf{E}[\cdot]$	Mathematical expectation
*	Ellipsis for the terms that are introduced by symmetry
$sym\{X\}$	$X + X^T$
$\star$	Matrices that are not relevant in the discussion
$l_2[0, \infty)$	The space of summable infinite sequence over $[0, \infty)$

2. Preliminaries

Consider the following discrete-time DMJSs:

$$\begin{cases} Ex(k+1) &= A(r_k)x(k) + B(r_k)\omega(k), \\ y(k) &= C(r_k)x(k) + D(r_k)\omega(k), \\ z(k) &= H(r_k)x(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $\omega(k) \in \mathbb{R}^\nu$ is the disturbance signal that belongs to $l_2[0, \infty)$, $y(k) \in \mathbb{R}^l$ is the output vector, and $z(k) \in \mathbb{R}^p$ is the signal to be estimated, respectively. The matrix $E \in \mathbb{R}^{n\times n}$ is singular with $\text{rank}(E) = r \le n$. For every $i \in \mathcal{J} = \{1, 2, \ldots, S\}$, $r_k = i$, we denote $A(r_k) = A_i$, $B(r_k) = B_i$, $C(r_k) = C_i$, $D(r_k) = D_i$, $H(r_k) = H_i$. A_i, B_i, C_i, D_i and H_i are known compatible dimension constant matrices. Switching law $\{r_k, k \ge 0\}$ is a discrete-time Markov stochastic process taking values in a finite state space $\mathcal{J} = \{1, 2, \ldots, S\}$, the evolution of $\{r_k, k \ge 0\}$ is governed by the following transition probabilities:

$$\Pr\{r_{k+1} = j | r_k = i\} = \mu_{ij},$$

where $\mu_{ij} \ge 0, \forall\ i, j \in \mathcal{J}$ with $\sum_{j=1}^S \mu_{ij} = 1$.

In this paper, we study the state estimation for system (1) over a lossy network, where the packet losses and asynchronous modes are simultaneously considered in the communication link between the plant and the estimator. In this case, we adopt the compensation scheme in [31] to cope with the effect of the packet losses:

$$\hat{y}(k) = (1 - \delta_k)y(k) + \delta_k\hat{y}(k-1). \tag{2}$$

The stochastic variable δ_k is a Bernoulli distributed white sequence with the following probability distribution laws:

$$\text{Prob}\{\delta_k = 1\} = \mathbf{E}\{\delta_k\} = \delta,$$
$$\text{Prob}\{\delta_k = 0\} = 1 - \mathbf{E}\{\delta_k\} = 1 - \delta,$$

where $\delta \in [0, 1]$ is a known constant.

Remark 1. In (2), the stochastic variables δ_k represents the possibility of occurring networked induced packet losses. Specifically, if $\delta_k = 0$, the communication is perfect, while $\delta_k = 1$ corresponds to the packet is lost in the measurement channel. Different from [33,36,37], the previous measurement data will be used as the observation data if the current measurement data packet is lost during transmissions in our paper.