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Discrete time domain analysis of chaos-based wireless communication systems with imperfect sequence synchronization

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ABSTRACT

Problem of sequence synchronization in chaos-based direct sequence spread-spectrum (DS-SS) systems and code division multiple access systems (CDMA) has been widely investigated. However, no mathematical expressions have been derived in closed form for the bit error probability in these systems when they operate with the imperfect time synchronization (represented by a delay between the received and reference spreading sequence generated in the receiver). Precise derivatives for this bit error probability are necessary to quantify the effect of imperfect synchronization on the overall properties of the system. To implement a random delay between the received and the receiver reference sequence, all signals in this paper are represented in the discrete time domain. To represent finite and random discrete delays between the sequences (which occur in a limited interval) the Gaussian and uniform probability density functions are expressed in discrete form. Furthermore, due to the finite value of possible random discrete delays, the expressions of related truncated density functions are expressed in closed form. Following this approach, the expressions for the bit error probability in closed form for chaotic and random spreading sequences have been derived.

1. Introduction

This paper presents the theoretical analysis of a direct sequences spread-spectrum (DS-SS) system and a code division multiple access (CDMA) system in the case of imperfect sequence synchronization. The systems are analyzed for binary and non-binary spreading sequences. These systems, with assumed perfect synchronization, have been widely analyzed and related theoretical models developed for binary, chaotic [1–3] and random spreading sequences including interleaving techniques to mitigate fading in the channel [4]. In particular, systems based on the application of chaotic spreading sequences have been the subject of substantial interest in the last two decades [1–3, 5–15].

The algorithms for sequence synchronization were developed which were based, in principle, on the assumption that the synchronization is performed in two phases, i.e., the acquisition and the tracking phase [16–25]. Asynchronous CDMA systems have been theoretically analyzed and compared with classical systems in [26–29]. However, although the sequences can be very well synchronized, a random time delay between the sequence received and the locally generated reference sequence will remain, which will increase the bit error probability.

Substantial attention has been given to derive expressions for the bit error probability (BEP), P_e, or simulate related bit error rate (BER) curves, and then investigate the system's behavior from combined interference, fading and noise perspective. Most of the work presented in existing papers was based on the assumption that the communication channel in the system is a flat fading channel. Further analysis of the systems with a wide-band (WB) channel was done in [29–34], and lately extended to the WB discrete time systems [3]. Furthermore, the BEP expressions in the existing literature are derived assuming that the perfect synchronization between the received and locally generated spreading sequence is achieved. In this paper, a detailed analysis of binary and non-binary DS-SS and CDMA systems (chaos-based and noise-based, in particular) and the procedure of deriving expressions for the BEP, assuming imperfect sequences synchronization, are presented. The BEP expressions are derived for the general case and contain an additional parameter, called *the sequence factor*, which depends on the statistical characteristics of the spreading sequences. By changing this factor the BER expressions for the binary or non-binary systems can be straightforwardly obtained. This factor was proven to be one for binary sequences.

The analysis of DS-SS and CDMA systems with related contributions, assuming imperfect sequences synchronization, is performed in this paper under the following conditions. Firstly, the acquisition phase of synchronization is considered to be completed and

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the system is in the tracking phase [16,24]. Because the tracking phase deals with the synchronization of chips we had to introduce and define the finite duration of a chip, T_c. Moreover, due to the discrete time domain presentation of the sequences in the analysis (which is suitable for direct implementation in digital technology) the chip values are represented by S interpolation samples defined by the time intervals T_s inside the chip duration T_c, i.e., $S = T_c/T_s$, allowing *a* quantification of various delays inside the chips. It is important to note that this linear interpolation preserves the bandwidth of the chip sequence and the bandwidth of the noise generated in the system simulation. For this case, the BEP expressions are derived which are conditioned on the random delay.

Secondly, in the systems analyzed there is a random error in the chip synchronization, which is defined as the discrete delay between the received and locally generated reference sequence. This error is characterized by discrete density functions [35]. The spreading sequences can be binary or non-binary, like chaotic or noise-like sequences [36–38] which are widely applied in spread-spectrum communications [39].

The similar analysis of synchronization error was conducted in the case when interleavers are used in the system to mitigate the fading [40,41]. A similar phenomenon is reported in [42–47], where a random phase difference exists between the received signal and the locally generated carrier. This random phase was usually represented by a Gaussian [42,44] or Tikhonov [42–47] density function (the Gaussian and uniform density function are used as special cases of Tikhonov density function [43]). These densities are used to find the mean value of the bit error probability (BEP) that was conditioned on the phase random variable [42,43,45–47]. In these papers these density functions are assumed to be continuous and the interval of their possible values sometimes was beyond the interval of possible values of the phase error as explicitly noted in [44]. Accordingly, all derivatives and results obtained had additional error due to this assumption. This error could be eliminated by the truncation of these density functions as it is shown herein. However, this truncation changes the moments of the related density functions. For example, by reducing the truncation interval of the Gaussian density the variance becomes smaller tending to zero when that interval of random values tends to zero [35]. The presented analysis can be directly applied to investigate the sequence synchronization in chaos based communication system presented in [48], where the received signal needs to be synchronized with the locally generated chaotic signal.

To understand how the truncation effect can be mitigated, we assume that the random delays can take values in a limited interval between $-S$ and $+S$, where S is the duration of a chip. Therefore, due to this limited interval, the discrete truncated density functions are used to represent random delays. This issue is a particular focus of this paper, and related discrete density functions are used with the discretization interval that is equal to the sampling interval T_s of the discrete time chip interval T_c. It was noted that the discrete density functions have both the mean and variance which depend on S and on the mean and variance of the related non-truncated continuous density functions [35]. To get correct results during simulation of the system this fact had to be taken into account.

Thirdly, the derived discrete truncated density functions, (represented by the Dirac delta functions), permit the derivation of BEP expressions in closed form. It is shown that, the existing BEP expressions for chaotic systems with perfect synchronization can be obtained from these general expressions as special cases by equating the delay to zero.

Fourthly, the closed form BEP expressions, for both binary and non-binary spreading sequences and the assumed imperfect sequence synchronization, are derived for three different channel models: the additive white Gaussian noise (AWGN) channel, the channel with AWGN and Rayleigh fading and the channel with AWGN and Rayleigh fading when the fading is mitigated using chip interleavers.

Fifthly, the derived theoretical expressions for the BEP are investigated analytically and confirmed via simulations. These expressions exactly quantify the influence of imperfect synchronization on the BEP. It is also shown that the reduction of this BEP can be achieved by introducing chip interleaving technique.

The paper is composed of seven Sections. In Section II the communication system, including transmitter, receiver and the channel, is presented. A mathematical model of the system in the presence of AWGN and the expressions for bit error probability are presented in Section III. A system analysis for a noisy fading channel is presented in Section IV. Section V presents a procedure of deriving the bit error probability expressions for the case when an interleavers are used in the system. Simulation results that confirm theoretical findings are presented in Section VI. Conclusions are in Section VII.

2. System structure and operation

The basic structure of a multi-user system, which is the subject of the mathematical analysis in this paper, is presented in Fig. 1. The system is composed of four parts: a multi-user transmitter (Tx), a fading channel (FC), a receiver (Rx), and a synchronization circuit.

At the input of the system are users' message bits. The ith bit of gth user is denoted by $\gamma_i{}^g$, which can have one of two values randomly taken from the set $\{-1,+1\}$. The N users' message bits are spread by N orthogonal and independent spreading sequences. In binary case these sequences can be pseudo random m-sequences, Walsh or Gold sequences, and in the case of non-binary sequences they can be chaotic [1,3] and generated from N chaotic sequence generators with different initial conditions. The gth user transmitter signal is $s_{ti}^g = \gamma_i^g x_{ti}^g$, where $x_{ti}{}^g$ is the tth chip for gth user for the ith bit (no commas have been included between indices for simplicity and clarity), i.e. $x_{ti}{}^g$ is an element of the discrete spreading sequence $(x_{1i}{}^g, x_{2i}{}^g, \ldots, x_{2\beta i}{}^g)$ [3]. The spread signals of N users are added to get $s_{tiTx} = \sum_{g=1}^N s_{ti}^g = \sum_{g=1}^N \gamma_i^g x_{ti}^g$ and then chip interleaved using a block interleaver (IL) of 2βx2β size. After transmission through the channel the received signal, affected by the noise and flat fading, is $s_{tiRx} = \sum_{g=1}^N \alpha_t \gamma_i^g x_{ti}^g + \xi_{ti}$, where α_t is a fading factor that affects the tth interleaved chip and ξ_{ti} is a sample value of noise inside tth chip of the ith bit. The all users, chips are transmitted through the fading channel, which is characterized by the Gaussian noise with two-sided power spectral density $N_0/2$ and Rayleigh fading with its density function $f_\alpha(\alpha) = (\alpha/b^2)e^{-\alpha^2/2b^2}$, having the mean $\eta_a = b(\pi/2)^{1/2}$, the mean square value $\eta_{a2} = 2b^2$ and the variance $\sigma_a{}^2 = b^2(4-\pi)/2$. The received sequence, which was interleaved in the transmitter inerleaver (IL) block, is deinterleaved inside the receiver deinterleaver (DI) block before it was correlated with the locally generated sequence.

The received multi-user sequence is correlated by a local reference sequence by means of multiplication and addition. It is important to note that (in this paper) the local reference sequence is not assumed to be perfectly synchronized with the received multi-user sequence s_{tiRx}. This imperfect synchronization is expressed by the discrete delay τ in Fig. 1, as illustrated in Fig. 2 by presenting a hypothetical example of the received and the reference sequence for the gth user. The sequences are represented and processed in discrete time domain. Each chip is represented by S interpolated chip samples. Therefore, the received chip, correlated with the delayed reference chip $x_{ti\tau}^g$, can be expressed in this form $z_t = \sum_{s=1}^S s_{tiRx} x_{ti\tau}^g = \sum_{s=1}^S (\sum_{g=1}^N \alpha_t \gamma_i^g x_{ti}^g + \xi_{ti}) x_{ti\tau}^g$, where $x_{ti\tau}{}^g$ is the delayed sample of tth chip of the local reference sequence