Signal Processing 154 (2019) 207–216

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Asymptotically optimal one-bit quantizer design for weak-signal detection in generalized Gaussian noise and lossy binary communication channel

Guanyu Wang, Jiang Zhu*, Zhiwei Xu

Ocean College, Zhejiang University, Zhoushan, China

ARTICLE INFO

Article history:
Received 27 April 2018
Revised 28 July 2018
Accepted 4 September 2018
Available online 5 September 2018

Keywords:
Threshold optimization
Weak-signal detection
Quantization
Generalized gaussian noise

ABSTRACT

In this paper, quantizer design for weak-signal detection under arbitrary binary channel in generalized Gaussian noise is studied. Since the performances of the generalized likelihood ratio test (GLRT) and Rao test are asymptotically characterized by the noncentral chi-squared probability density function (PDF), the threshold design problem can be formulated as a noncentrality parameter maximization problem. The theoretical property of the noncentrality parameter with respect to the threshold is investigated, and the optimal threshold is shown to be found in polynomial time with appropriate numerical algorithm and proper initializations. In certain cases, the optimal threshold is proved to be zero. Finally, numerical experiments are conducted to substantiate the theoretical analysis.

1. Introduction

Signal estimation and detection from quantized data continues to attract attention over the past years [1–16]. In [1], a general result is developed and applied to obtain specific asymptotic expressions for the performance loss under uniform data quantization in several signal detection and estimation problems including minimum mean-squared error (MMSE) estimation, non-random point estimation, and binary signal detection. In [2], a distributed adaptive quantization scheme is proposed for signal estimation, where individual sensor nodes dynamically adjusts their quantizer threshold based on earlier transmissions from other sensor nodes. In [3], distributed parameter estimators based on binary observations along with their error-variance performance are derived in the case of an unknown noise probability density function (PDF). For the robust estimation of a location parameter, the noise benefits to maximum likelihood type estimators are investigated [4]. As a result, the analysis of stochastic resonance effects is extended for noise-enhanced signal and information processing. In [5], distributed detection of a non-cooperative target is tackled, and fusion rules are developed based on the locally-optimum detection framework. Recently, some variants of the classical signal estimation and detection model from quantized data are studied. One is that the unquantized observations are corrupted by combined multiplicative and additive Gaussian noise [6–8]. Another is called the unlabeled sensing where the unknown order of the quantized measurements causes the entanglement of desired parameter and nuisance permutation matrix [9,10]. In [11,12], the authors investigate the estimation problem under generalized Gaussian noise (GGN) and reveal the property of the Fisher information (FI). In addition, a systematic framework for composite hypothesis testing from independent Bernoulli samples is studied, and the comparison of detectors are made under one-sided and two-sided assumptions [13].

The threshold of the quantizer can be designed to improve the performance of estimation and detection [17–26]. In the early paper [17], two useful detection criteria are proposed, leading to the MMSE between the quantized output and the locally optimum nonlinear transform for each data sample. Later in [18], the optimal quantized detection problem is considered for the Neyman-Pearson, Bayes, Ali-Silvey distance, and mutual (Shannon) information criteria, and it is shown that the optimal sensor decision rules quantize the likelihood ratio of the observations. In the design of quantized detection systems, the optimal test is shown to employ a nonrandomized rule under certain conditions, which considerably simplifies the design [19]. In [20], it is shown that given a particular constraint on the fusion rule, the optimal local decisions which minimize the error probability amount to a likelihood-ratio test (LRT). In addition, a design example with a binary symmetric channel (BSC) is given to illustrate the usefulness of the result in obtaining optimal threshold for local sensor observations. In [21], the maximin asymptotic relative efficiency (ARE) criterion is proposed to optimize the thresholds, and the improvement of

---

* Corresponding author.
E-mail addresses: guanyuwang@zju.edu.cn (G. Wang), jiangzhu16@zju.edu.cn (J. Zhu), xuzw@zju.edu.cn (Z. Xu).

https://doi.org/10.1016/j.sigpro.2018.09.005
0165-1684/© 2018 Elsevier B.V. All rights reserved.

estimation performance is demonstrated in distributed systems. Utilizing the asymptotic performance of the one-bit generalized likelihood ratio test (GLRT) detector, the optimal threshold is proven to be zero under Gaussian noise and a BSC [22]. The quantizer design is also analyzed under the GGN [23,24]. In [23], the problem is considered under the error-free channel, and the optimal threshold is only plotted without theoretical justification. The BSC is also included in the successional studies [24–26]. In [24], zero is shown to be the optimal threshold when the shape coefficient is less than or equal to two and a good (sub-optimal) choice when the shape coefficient is larger than two. Analogously, zero is employed as a good choice in the generalized Rao test [25]. For generalized locally optimum detectors, the threshold optimization is re-formulated as a maximization problem in terms of the local false-alarm probability, which can be easily evaluated via one-dimension numerical search [26].

1.1. Related work

The most related work to ours is [22–24]. All the three articles focus on the problem of quantizer threshold design but in difference noise and channel models. In [22], the optimal threshold is shown to be zero in the case of Gaussian noise and BSC. In [23], under the GGN and lossless channel, the authors propose effective thresholds without any rigorous analysis. In [24], zero threshold is chosen as a suboptimal choice (not too bad) for the GGN and BSC case. In this paper, we extend [22–24] to general settings, more explicitly, we extend the Gaussian noise in [22] to the GGN and the BSC in [23,24] to arbitrary (symmetric/asymmetric) binary channels.

Besides, the present paper works on the property of the FI in the case of GGN, which is also related to the works in [11,12]. Hence it is crucial to make a qualitative comparison with the results obtained in [11,12]. In [11], the authors focus on the property of the FI under symmetrically distributed noise and provide a specific condition under which zero threshold is a local minimum of the FI. According to the condition, zero point is the optimal threshold for the GGN ($1 \leq \beta \leq 2$) and Cauchy cases but locally the worst choice for the GGN ($\beta > 2$) and uniform/Gaussian cases. In contrast, we study more than the property of the FI at one point but its monotonicity in the domain, which aims to obtain the precisely optimal threshold. In [12], the authors analyze the FI of non-uniform and multi-bit quantized data. By minimizing the approximation of the information loss caused by quantization, the authors obtain the optimal threshold under GGN and Student's-distributions (STD). The validity of the approximations are verified for the special cases, i.e., GGN ($\beta = 2$) and STD ($\beta = 1$). Compared to [12] focusing on the approximation of the optimal threshold for multi-bit quantization, we study the one-bit quantization problem and show that the optimal threshold can be efficiently found in polynomial time.

1.2. Main contributions

The main contribution of this paper is to address the threshold design problem under GGN in the arbitrary binary channel. We extend [22–24] to general settings, more explicitly, we extend the Gaussian noise in [22] to the GGN and the BSC in [23,24] to arbitrary (symmetric/asymmetric) binary channels. This general model can model the scenario where the sensors receive signals corrupted by GGN and transmit the quantized signals to the FC via an additive white Gaussian noise (AWGN) channel, and the FC quantizes the received signals into a single bit, which can be abstracted as the asymmetric binary channel, as shown in Fig. 1. The upper right dashed block of Fig. 1 can be abstracted as an ideal binary asymmetric channel [27]. The meaningfulness of extending the Gaussian noise to the GGN is twofold. From the analytic point of view, the generalized Gaussian distribution is flexible, encompassing a wide range of distribution functions [28]. From a practical standpoint, it models a variety of physical phenomenon in the ocean acoustic environments [29,30]. Therefore, the model in this paper is more general and the practical applications will benefit from the current assumptions.

Under the weak-signal assumption, the thresholds can be optimized via maximizing the noncentrality parameter. Unfortunately, it is difficult to prove the theoretical properties of the noncentrality parameter function with respect to the threshold. We novelly propose a simplified function whose sign is the same as that of the first derivative of the noncentrality parameter function. Consequently, we rigorously prove the theoretical properties of the noncentrality parameter function with respect to the threshold indirectly. Then we prove that for arbitrary binary channel, the optimal threshold can be found in polynomial time via appropriate numerical algorithm with proper initializations. In certain cases, we prove the optimal threshold to be zero.

1.3. Organization

The paper is organized as follows. In Section 2, the weak-signal detection problem is described, and preliminary materials including both maximum likelihood (ML) estimation and parameter tests are introduced. Section 3 states the main results of the quantizer design. In addition, an algorithm to calculate the optimal threshold is proposed. In Section 4, numerical experiments are conducted to substantiate the theoretical analysis. The conclusions are presented in Section 5. Finally, the related functions and the proof of propositions are presented in Appendix A.

1.4. Notation

Let $1\{\cdot\}$ denote an indicator function which produces 1 if the argument is true and 0 otherwise. Let χ_n^2 and $\chi_n'^2(\lambda_Q)$ be a central chi-squared PDF with n degrees of freedom and a noncentral chi-squared PDF with n degrees of freedom and noncentral parameter λ_Q, respectively. Let $\Gamma(\cdot)$ denote a gamma function. For a random matrix $\mathbf{U}$, $[\mathbf{U}]_{ij}$ denotes its element at ith row and jth column, and $\mathrm{E}_{\mathbf{U}}[\cdot]$ denotes the expectation taken with respect to it. For a random GGN w, α^{-1} denotes its scale parameter, β denotes its shape parameter, $f(w)$ denotes its PDF, and $F(w)$ denotes its cumulative distribution function (CDF). In Section 2 and 3, functions $G(x, q_0, q_1)$, $G(x)$, $M(x)$, $m_1(x)$, $m_2(x)$, $m_3(x)$, points x_0, x_0', x_1, x_2 and superscripts are introduced for better presentation. For an optimized variable, the superscript $*$ denotes its optimal value. For a function (excluding point x_0'), the superscripts $'$ and $''$ separately denote its first and second derivatives.

2. Problem setup

In this section, a weak-signal detection model is described. First, we introduce GLRT and Rao tests on the basis of ML estimation. For quantitative analysis, we use an approximation of parameter tests and show that the detection performance can be characterized by the noncentral parameter which is a function of quantizer thresholds. As a result, the quantizer thresholds design problem is formulated as a noncentral parameter maximization problem.

2.1. Weak-signal detection model

We consider a binary hypothesis testing problem, in which N distributed sensors in a wireless sensor network (WSN) are utilized K times to generate noisy observations. Those observations are quantized with different thresholds, and then used to detect