Signal Processing 154 (2019) 232–237

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Bias analysis in Kalman filter with correlated mode mismatch errors

Wenji Zhang*, Balasubramaniam Natarajan

Department of Electrical and Computer Engineering, Kansas State University, Manhattan, KS 66506, United States

ARTICLE INFO

Article history:
Received 30 March 2018
Revised 28 August 2018
Accepted 5 September 2018
Available online 5 September 2018

Keywords:
Markov jump linear system
Kalman filter

ABSTRACT

In this paper, we investigate the impact of inaccurate mode knowledge on a mode-based Kalman filter used for state estimation in a Markov jump linear system (MJLS). Specifically, we consider correlated mode mismatch errors that can effectively capture the influence of communication network errors or cyber-attacks in a cyber-physical system. Interpreting the bias dynamics as a transformed switching system, we derive, for the first time, an algebraically solvable sufficient condition (with respect to mode mismatch error probabilities) that guarantees convergence of the bias. The theoretical result, validated via simulations, provides insights on the fidelity of discrete state knowledge required for maintaining the continuous state estimate quality.

1. Introduction

Stochastic hybrid systems (SHS) have been effectively used to model dynamical systems that experience the interaction of discrete and continuous dynamics with uncertainty. Markov jump linear system (MJLS) [1] is a subclass of SHS where the continuous states evolve linearly and discrete modes switch following a Markov chain. MJLS has been used in modeling of cyber-physical systems (CPS) such as microgrid [2], networked control systems [3], etc. State estimation is critical for MJLS if the states are not directly accessible. The quality of state estimation impacts further system analysis and the design of an appropriate control strategy. State estimation in MJLS is dominated by Kalman filter based algorithms. It has been shown that if the discrete states are observable (known), the Kalman filter is an optimal estimator [1]. When the discrete states are not available, the optimal estimator is obtained from a bank of Kalman filters, such as the well-known interacting multiple model (IMM) algorithm [4] and multiple model adaptive estimation (MMAE) algorithm [5]. In terms of performance analysis for hybrid system estimation algorithms, due to the complexity of hybrid strategies, there are limited efforts that have been made in this field. To our best knowledge, only few works have studied the performance of hybrid estimation algorithms [6–8]. Specifically, [6–8] focus on stability of IMM algorithm[9]. studies the effect of mismodeling in Kalman filter for non-hybrid system settings and it derives the mean and covariance matrix for residuals of a mismodeled Kalman filter without analyzing the stability or convergence of the residual.

In this work, we take a fresh perspective on performance analysis for MJLS with mode mismatch occurrences. We consider cases where we have information on the discrete states but the information is inaccurate (mode mismatch). Specifically, we focus on correlated mode mismatch errors and the correlations are across different modes. In this situation, instead of implementing a bank of Kalman filters at the cost of the exponentially increasing memory and computing time, we can treat the known discrete states as the true state and conduct the estimation via only one Kalman filter. The inaccurate mode information will introduce a bias to the estimator with the error covariance remaining bounded [10]. The focus of this work is to derive conditions under which the bias dynamics is statistically convergent. The notion of correlated mode mismatch errors can efficiently capture communication-link failures and spatially correlated cyber-impairments in CPSs. A motivating example is presented below:

1.1. Motivating example

Network topology error is a typical problem in a smart grid [11]. Consider a conceptual smart grid as shown in Fig. 1. The system consists of a bank of photovoltaic (PV) panels, three home loads, an electric vehicle and the main electricity grid. As discussed in [12], a smart grid can be well described using a framework of SHS because of the interaction between probabilistic elements and discrete and continuous dynamics. For example, the underlying analog/continuous variables are power consumption, bus voltage, etc. The discrete behaviors are captured by status of switches. The status of switches determines the network topology of the smart grid. As shown in Fig. 1, there could be communication link failures near switches S_1 and S_2 which in turn result in erroneous

* Corresponding author.
E-mail addresses: wenjizhang@ksu.edu (W. Zhang), bala@ksu.edu (B. Natarajan).

https://doi.org/10.1016/j.sigpro.2018.09.008
0165-1684/

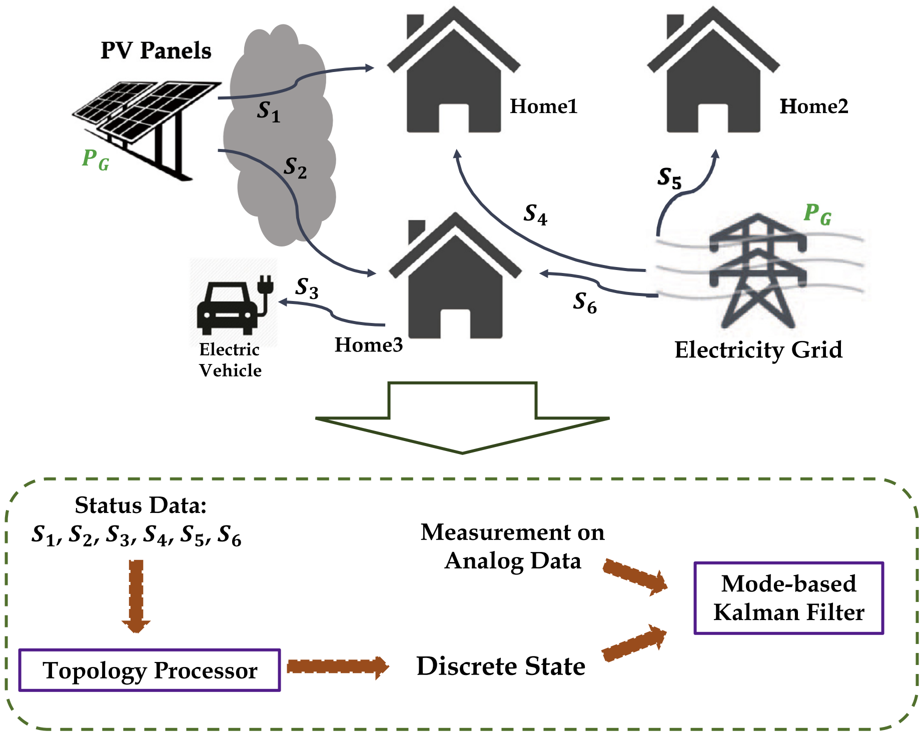

Fig. 1. Motivating example: spatial correlated cyber-effects in a smart grid.

status information reported to the topology processor. Consequently, the incorrect status leads to a mode mismatch (which could be spatially correlated as network impairments can impact multiple switch status information) that in turn will introduce a bias in the state estimation for analog variables. For smart grids and many other CPSs that experience similar affects, this work derives conditions for statistical convergence of the bias dynamics of a mode-based Kalman filter estimate.

1.2. Contributions

As stated earlier, mode mismatch is a typical problem in MJLS state estimation. For a general class of SHS, review of the recent literature reveals that there is limited prior work that provides a rigorous performance analysis of SHS estimation techniques [6–8]. Both [6,7] focus on the convergence of residual for hybrid estimation algorithms IMM. A recent research [8] studies sufficient conditions on error covariance of IMM algorithm for MJLS state estimation. All the above mentioned research works focus on IMM algorithm (composed of multiple mode-based Kalman filters). However, for a general hybrid system estimation setting, the performance of mode-based Kalman filter with presence of mode mismatch errors has not been addressed. In this regard, the contributions of this work can be summarized as follows:

- This paper studies the impact of mode mismatch in a mode-based Kalman filter for MJLS estimate. As an extension of our prior work [10], we consider the case of correlated mode mismatches that can capture spatially correlated cyber-impairments in practical applications.
- This paper derives sufficient conditions under which the bias resulting from mode mismatches is statistically convergent. The condition is related to mode mismatch probabilities and it provides guidance on the fidelity of discrete state information needed to sustain the quality of the Kalman filter estimate.
- The novelty of this paper lies in modeling the bias dynamics as a transformed switching system. By leveraging existing results in stability analysis for switching system, we obtain conditions for the convergence of bias. Furthermore, for the first time, we are able to derive an algebraically solvable condition in terms of the mode mismatch probabilities that guarantees the statistical convergence of the bias.

The rest of the paper is organized as follows: The system model is introduced in Section II. In Section III, we derive the dynamics of bias and present conditions that guarantee its statistical convergence. A numerical example is presented in Section IV to validate the theoretical result. We conclude this work and discuss future directions in Section V.

2. Preliminaries

2.1. Notations

We use normal face to define scalars; Bold face to define vectors (lower case) or matrices (upper case); $\mathbf{0}$ represents zero-vector and $\mathbf{I}$ represents identity matrix; $\mathbb{R}$ denotes the space of real number; $\mathbf{A}\otimes\mathbf{B}$ represents the Kronecker product of two matrices; $\rho(\cdot)$ denotes the spectral radius of any square matrix; $\|\cdot\|$ represents 2-norm of a matrix or a vector; $[\cdot]'$ and $[\cdot]^{-1}$ denote transpose of a matrix (or a vector) and inverse of a matrix respectively; $|\mathcal{C}|$ denotes the number of elements in the set $\mathcal{C}$. $\mathbb{E}(\cdot)$ represents expectation and $\mathbb{P}(\cdot)$ represents probability measure.

2.2. System model

We consider a discrete-time MJLS with a finite discrete state space denoted as $\mathcal{Q} = \{q_1, \cdots, q_d\}$. The discrete state transitions are modeled via a time-homogeneous Markov chain. Let $\delta_k : k \to \mathcal{Q}$ be a switching signal. Then the transition probability corresponds