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Short communication

1-bit compressive sensing with an improved algorithm based on fixed-point continuation

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ABSTRACT

In this paper, 1-bit compressive sensing with improved reconstruction algorithms based on the fixed-point continuation (FPC) method is investigated. By introducing appropriate modifications to the conventional FPC-ℓ_2 algorithm, the improved algorithms enjoy several advantages simultaneously. First, the prior knowledge of sparsity level is not required. Second, with a one-sided ℓ_1-norm to impose consistency, the performance of the proposed FPC-ℓ_1 algorithm offers better performance than the previous FPC-ℓ_2 algorithm. Third, by incorporating an adaptive outlier pursuit (AOP) to the FPC-ℓ_1 algorithm, the resulting FPC-AOP-ℓ_1 algorithm achieves improved robustness against noise. Numerical results are provided to demonstrate the effectiveness and superiority of the proposed algorithm.

1. Introduction

Compressive sensing (CS) is known to be a powerful signal acquisition method in which high dimensional signals can be estimated from a set of relatively less measurements [1]. Taking the sparsity of the signal structure into account, CS successfully reduces the sampling-rate requirement to stably recover sparse signals. Under the CS framework, we acquire a signal $\mathbf{x} \in \mathbb{R}^N$ via the linear measurements

$$\mathbf{y} = \mathbf{\Phi x} \tag{1}$$

where $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is the sampling matrix with $M \ll N$ and satisfies the restricted isometry property (RIP) [2], $\mathbf{x}$ has only K non-zero coefficients, and $\mathbf{y} \in \mathbb{R}^M$ denotes the acquired measurements.

In practice, the quantization process after the measurement is generally unavoidable, which means that CS measurements must be quantized from a real value to a discrete value. Many reconstruction techniques which address CS quantization have been proposed, and in general these techniques model the quantization error as additive measurement noise. Recent advances in Quantized-CS theory have led to an increasing interest for studying the 1-bit quantization case [3,4]. As an extreme case of CS, 1-bit CS preserves only the sign information of the measurements, which significantly reduce the complexity as well as cost of hardware implementation (the quantizer is just a comparator). Besides, 1-bit CS is more robust against nonlinear distortions, and in certain situations it can perform even better than conventional methods [5]. Since only the sign information is recorded, the performance of 1-bit CS relies on the number of measurements. The number of measurements can be larger than the dimension of the signal, which is different from regular CS.

In 1-bit CS, measurements of the N-dimension signal $\mathbf{x}$ can be written as

$$\mathbf{y} = sign(\mathbf{\Phi x}) \tag{2}$$

where $\mathbf{y} = [y_1, y_2, \dots, y_M]^T \in \{-1, 1\}^M$ consists of the binary measurements, and the operator $sign(\cdot)$ is the sign function applied to component-wise on $\mathbf{\Phi x}$, i.e., $sign(z)$ equals 1 if $z > 0$ and -1 otherwise for any $z \in \mathbb{R}$. Since only the signs of real-valued measurements are preserved, scaling $\mathbf{x}$ will not make changes on the measurements. In other words, the amplitude information are lost due to the 1-bit quantization, the norm of $\mathbf{x}$ cannot be recovered from the binary measurements. To this end, a unit energy constraint $\|\mathbf{x}\|_2 = 1$ can be imposed. The meaning of signal recovery can be explained as finding the optimal value partitioned by random hyperplanes. With the unit energy constraint used, the feasible set is reduced from the N-dimension space to the unit sphere, and consequently the speed and the performance of reconstruction are both improved.

One of the earliest reconstruction algorithms for 1-bit CS was proposed by Boufounos and Baraniuk in 2008 in [6], where a one-sided ℓ_2-norm is utilized to impose consistency and a fixed-point continuation (FPC) algorithm [7] is employed for reconstruction. The consistent reconstruction means that the quantized measurement y_i, $i = 1, \dots, M$, have the same sign information with the corresponding recovered measurements $[\mathbf{\Phi\hat{x}}]_i$, where $\mathbf{\hat{x}}$ is the recov-

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ered signal. While the FPC-based algorithm is effective in adapting some optimization methods to the unit norm constraint, it exhibits relatively poor performance. Nevertheless, the FPC-based algorithm does not require sparsity level as prior knowledge, which is a desired feature in practical applications. In order to improve the reconstruction performance, a variety of 1-bit compressive sensing algorithms have been proposed, such as matching sign pursuit (MSP) [8], binary iterative hard thresholding (BIHT) [9] and History algorithm [10]. Although achieving good performance, these algorithms need the sparsity level as an input.

Another challenge to 1-bit CS is the robustness problem in noisy environments. In noisy scenarios, the binary measurements are randomly perturbed, and the so-called sign-flips seriously degrade recovery performance. Towards this end, several approaches have been proposed. For instance, Yan introduced a modified version of BIHT, referred to as adaptive outlier pursuit (AOP), which is robust against bit flips [11]. Movahed proposed a noise-adaptive renormalized fixed point iteration (NAFRPI) algorithm that embedded the AOP technique into the FPC algorithm [12]. Plan developed a convex optimization method regarding noisy 1-bit compressed sensing [13]. It was demonstrated that the algorithm can work with a general notion of noise and error for both exactly and approximately sparse signals. Based on Plan's model, Zhang et al. developed an efficient Passive algorithm with closed-form solution, which improves the recovery performance [14]. Note that these algorithms either require the knowledge of sparsity level or exhibit limited performance.

Motivated by the limitations of the aforementioned approaches, in this paper, improved algorithms are proposed for 1-bit CS by modifying the FPC-ℓ_2 algorithm. The main contribution of this paper is to provide a simple but valid algorithm to achieve state-of-the-art performance when sparsity is unknown. The proposed algorithms utilize a one-sided ℓ_1-norm to impose consistency, and the performance of the proposed algorithms can be remarkably improved compared to the existing algorithms. More importantly, the proposed algorithms do not require the knowledge of sparsity level. Although the one-sided ℓ_1-norm is also suggested in BIHT-ℓ_1 algorithm, the one-sided ℓ_1-norm has not appear in any sparsity-free algorithms, to our knowledge. The sparsity-free feature will be greatly useful in real applications. In addition, robustness against noise can be well guaranteed based on an AOP scheme. The effectiveness and superior performance of the proposed algorithm are demonstrated by numerical results.

2. Problem formulation and FPC-ℓ_2 algorithm

Following the typical CS framework, in order to recover the signal from 1-bit measurements, we enforce sparsity by minimizing the ℓ_1-norm of the reconstructed signal, i.e., $\|\mathbf{x}\|_1$. Moreover, with the 1-bit quantization, the signs of the quantized measurement and real value are the same. In other words, we have

$$\mathbf{y} \odot \mathbf{\Phi x} \succeq \mathbf{0} \tag{3}$$

where $\odot$ and $\succeq$ denote element-wise product and element-wise inequality, respectively, and $\mathbf{0}$ is a vector with all entries equal to zero. Note that (3) can also be written as written as $\mathbf{Y\Phi x} \succeq \mathbf{0}$, where $\mathbf{Y} = diag(\mathbf{y})$ denotes a diagonal matrix with the measurement signs on the main diagonal. For notational simplicity, let us define $\mathbf{Z}$ as

$$\mathbf{Z} \triangleq \mathbf{Y\Phi}. \tag{4}$$

As a result, the following problem can be formulated for signal reconstruction:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|_1 \\ s.t. \quad & \mathbf{Zx} \succeq \mathbf{0} \\ & \|\mathbf{x}\|_2 = 1. \end{aligned} \tag{5}$$

It is seen that besides the consistency constraint, an additional energy constraint $\|\mathbf{x}\|_2 = 1$ is imposed for the purpose of avoiding the trivial solution $\mathbf{x} = \mathbf{0}$. Obviously, the problem (5) is nonconvex. To this end, Boufounos and Baraniuk [6] proposed to relax the problem by introducing a one-sided quadratic cost function $f(x)$ as

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|_1 + \lambda \sum_i f([\mathbf{Zx}]_i) \\ s.t. \quad & \|\mathbf{x}\|_2 = 1 \end{aligned} \tag{6}$$

where λ is a relaxation parameter, $[\cdot]_i$ denotes the ith entry of a vector, and $f(x)$ is given by

$$f(x) = \begin{cases} 0, & \text{if } x \geq 0 \\ x^2/2, & \text{otherwise} \end{cases} \tag{7}$$

Note that $\sum_i f([\mathbf{Zx}]_i)$ is a barrier function, as λ tends to infinity, solutions to the problems (5) and (6) will be the same. In order to solve the problem (6), the FPC algorithm [7] can be employed by computing the one-sided ℓ_2-norm penalty on the unite sphere $\|\mathbf{x}\|_2 = 1$ followed by a renormalization step [6]. This results in the FPC-ℓ_2 algorithm.

It is seen that the FPC-ℓ_2 algorithm imposes consistency by using a one-sided quadratic penalty function $f(x)$. Naturally, one might expect that a different penalty function would lead to different performance. This leaves us the possibility to improve the performance by choosing a more appropriate function $f(x)$. Furthermore, in practice, during the acquisition as well as transmission process, measurements are always contaminated by noise, which would make the 1-bit measurements (signs) flipped, and hence, degrade the reconstruction performance. Motivated by these facts, modifications will be made to the FPC-ℓ_2 algorithm so as to achieve improved and robust reconstruction.

3. Proposed FPC-ℓ_1 algorithms

3.1. Noiseless 1-bit measurements

As mentioned earlier, the existing FPC-ℓ_2 algorithm [6] enforces consistency by using a one-side quadratic penalty function (7) and corresponding objective $\sum_i f([\mathbf{Zx}]_i)$, which can be expressed as a one-sided ℓ_2-norm as $\sum_i f([\mathbf{Zx}]_i) = \frac{1}{2}\|[\mathbf{Zx}]_-\|_2^2$, where $[\cdot]_-$ denotes the negative function, i.e., $[x]_- = (x - |x|)/2$. As a matter of fact, in order to impose consistency, we can also use the following one-sided linear function:

$$f(x) = \begin{cases} 0, & \text{if } x \geq 0 \\ |x|, & \text{otherwise} \end{cases} \tag{8}$$

and the corresponding one-sided ℓ_1-norm objective function

$$\sum_i f([\mathbf{Zx}]_i) = \|[\mathbf{Zx}]_-\|_1 \tag{9}$$

More importantly, as discussed in [9], one-sided ℓ_1-norm (related to hinge-loss) is superior to the one-sided ℓ_2-norm (related to square loss) in the context of binary classification, which also enforces the same consistency function as in (3). This motivates us to employ the one-sided ℓ_1-norm for performance improvement. Specifically, the following problem is proposed for 1-bit CS reconstruction:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|_1 + \lambda \|[\mathbf{Zx}]_-\|_1 \\ s.t. \quad & \|\mathbf{x}\|_2 = 1 \end{aligned} \tag{10}$$

To solve the problem (10), the FPC algorithm can be employed. The FPC algorithm was originally proposed to solve the optimization problems with the following form

$$\min_{\mathbf{x}} \quad \|\mathbf{x}\|_1 + \lambda \varphi(\mathbf{x}) \tag{11}$$