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The meniscus on the outside of a circular cylinder: From microscopic to macroscopic scales

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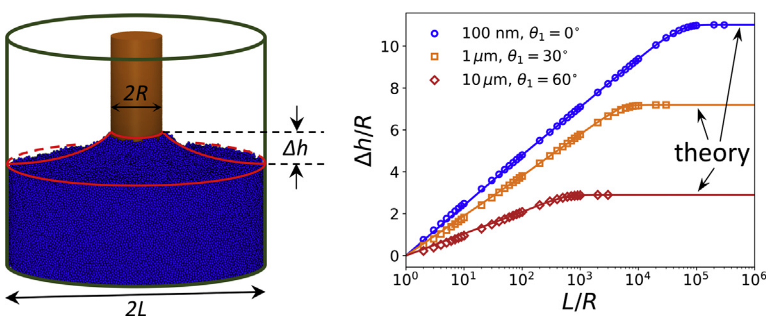
GRAPHICAL ABSTRACT



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ABSTRACT

We systematically study the meniscus on the outside of a small circular cylinder vertically immersed in a liquid bath in a cylindrical container that is coaxial with the cylinder. The cylinder has a radius R much smaller than the capillary length, κ^{-1}, and the container radius, L, is varied from a small value comparable to R to ∞. In the limit of $L \ll \kappa^{-1}$, we analytically solve the general Young-Laplace equation governing the meniscus profile and show that the meniscus height, Δh, scales approximately with $R\ln(L/R)$. In the opposite limit where $L \gg \kappa^{-1}$, Δh becomes independent of L and scales with $R\ln(\kappa^{-1}/R)$. We implement a numerical scheme to solve the general Young-Laplace equation for an arbitrary L and demonstrate the crossover of the meniscus profile between these two limits. The crossover region has been determined to be roughly $0.4\kappa^{-1} \lesssim L \lesssim 4\kappa^{-1}$. An approximate analytical expression has been found for Δh, enabling its accurate prediction at any values of L that ranges from microscopic to macroscopic scales.

1. Introduction

A liquid meniscus as a manifestation of capillary action is ubiquitous in nature and our daily life. For example, its formation and motion play critical roles in water uptake in plants [1]. Capillary adhesion due to the formation of menisci between solid surfaces makes wet hair to stick together and allows kids to build sandcastles [2]. Menisci are also involved in many technologies and industrial processes [3] such as meniscus lithography [4], dip-pen nanolithography [5], dip-coating (Langmuir-Blodgett) assembly of nanomaterials [6–8], meniscus-mediated surface assembly of particles [9], meniscus-assisted solution printing [10], etc.

A meniscus system frequently discussed in the literature is the one formed on the outside of a circular cylinder that is vertically immersed in a liquid bath. One application of this geometry is the fabrication of fiber probes by chemical etching [11]. A cylinder

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with radius at the nanometer scale has also been attached to the tip of an atomic force microscope to perform nano-/micro-Whilhemy and related liquid property measurements [12]. The shape of the meniscus is governed by the Young-Laplace equation [13]. Extensive studies have been reported for the scenario where the liquid bath is unbound and the lateral span of the liquid-vapor interface is much larger than the capillary length of the liquid [14–19]. Different methods have been applied in these studies, including numerical integration [15,16] and analytical approaches such as matched asymptotic expansions [17–19] and hodograph transformations for cylinders with complex shapes [19]. An approximate formula has been derived for the meniscus height, which depends on the radius of the cylinder and the contact angle of the liquid on the cylinder surface [14,17]. The meniscus exerts a force that either drags the cylinder into or expels it from the liquid depending on if the contact angle is acute or obtuse. A recent study of the meniscus rise on a nanofiber showed that the force on the nanofiber highly depends on the lateral size of the liquid-vapor interface if this size is smaller than the capillary length [20].

In this paper we consider a geometry as sketched in Fig. 1 where a small circular cylinder vertically penetrating a liquid bath that is confined in a cylindrical container. With the cylinder and the container being coaxial, the system has axisymmetry that enables certain analytical treatments. By fixing the contact angle on the surface of the container to be $\pi/2$, we have a meniscus that systematically transits from being laterally confined to unbound, when the size of the container is increased. For such a system, the meniscus profile is governed by the general Young-Laplace equation that was first studied by Bashforth and Adams more than a century ago [21]. This equation has been discussed in various systems including liquid in a tube [22], sessile and pendant droplets [23,24] and a capillary bridge between two spheres [25].

In the limit where the size of the cylindrical container is much smaller than the capillary length, the gravitational term in the Young-Laplace equation can be neglected and the equation becomes analytically solvable. Solutions have been reported for various capillary bridges between solid surfaces [26–28] and tested with molecular dynamics simulations [29,30]. We have obtained a solution for the meniscus in Fig. 1 based on elliptic integrals when the lateral size of the meniscus is small and found that the meniscus height depends on the container size logarithmically. We further numerically solve the full Young-Laplace equation for an arbitrary container size and find that the meniscus height approaches an upper limit found in some early work when the lateral span of the interface is much larger than the capillary length [14,16,17]. Finally, we find an approximate expression of the meniscus height on the cylinder that is applicable to any lateral size of the liquid-vapor interface. This work is the basis of a related work on the wetting behavior of particles at a liquid-vapor interface [31], where the theoretical results presented here are applied to study the detachment of a spherical particle from a liquid bath.

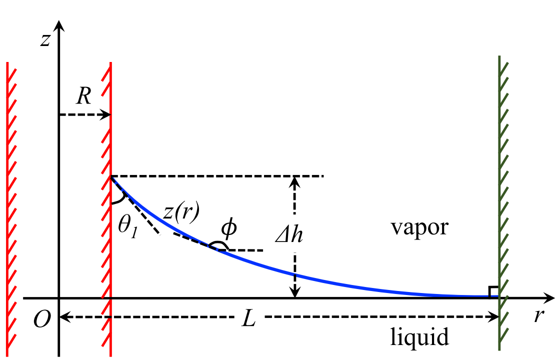

Fig. 1. A rising meniscus on the outside of a circular cylinder vertically immersed in a liquid bath confined in a cylindrical container that is coaxial with the cylinder.

2. Theoretical considerations

2.1. General equation of the meniscus shape

The geometry of the system considered in this paper is sketched in Fig. 1. A circular cylinder with radius R is immersed in a liquid bath confined in a cylindrical wall with radius $L > R$. The cylinder and the wall are coaxial and the system is thus axisymmetric. The shape of the meniscus in this ring-shaped tube is determined by the surface tension of the liquid, the contact angles on the two surfaces, and possibly gravity. Our interest is to examine the crossover from the case where $L - R$ is small to the case where the cylinder is immersed in a liquid bath with an infinite lateral span. Since in the latter limit the liquid-vapor interface is flat at locations far away from the cylinder, we will set the contact angle on the wall to be $\pi/2$. Then a meniscus will rise (depress) on the outside of the cylinder if the contact angle on its surface, θ_1, is smaller (larger) than $\pi/2$. The case where $\theta_1 = \pi/2$ is trivial with the liquid–vapor interface being flat everywhere. Here we focus on the case with $\theta_1 < \pi/2$, where a meniscus rises on the cylinder and generates a force to pull the cylinder into the liquid bath. However, the final results on predicting the meniscus height also apply to the case where $\theta_1 > \pi/2$.

The equilibrium shape of the meniscus is governed by a form of the Young-Laplace equation studied by Bashforth and Adams before [21],

$$\frac{z''}{(1+z'^2)^{3/2}} + \frac{z'}{r(1+z'^2)^{1/2}} = \frac{\Delta p}{\gamma} + \frac{\Delta \rho g z}{\gamma}, \tag{1}$$

where $z(r)$ is the meniscus height at distance r from the central axis of the cylinder, $z' \equiv \frac{dz}{dr}$, $z'' \equiv \frac{d^2z}{dr^2}$, Δp is the pressure jump from the vapor to the liquid phase at $r = L$ and $z = 0$, γ is the surface tension of the liquid, $\Delta\rho \equiv \rho_l - \rho_v$ is the difference of the liquid and vapor densities, and g is the gravitational constant. A brief derivation of this equation is provided in Appendix A. In the following discussion, we use a water-air liquid interface at 25 °C as an example, for which $\gamma \approx 0.072$ N/m and $\Delta\rho \approx 10^3$ kg/m^3.

To facilitate discussion, we define $2\widetilde{H} \equiv \frac{\Delta p}{\gamma}$ and $\kappa^2 \equiv \frac{\Delta\rho g}{\gamma}$, i.e., $\kappa^{-1} = \sqrt{\frac{\gamma}{\Delta\rho g}}$ is the so-called capillary length, which is a characteristic length scale of the problem. For water at 25 °C, $\kappa^{-1} \approx 2.7$ mm. Eq. (1) can then be made dimensionless via a variable change

$$x \equiv \kappa r, \quad y \equiv \kappa z. \tag{2}$$

The result is the following nonlinear differential equation

$$\frac{y''}{(1+y'^2)^{3/2}} + \frac{y'}{x(1+y'^2)^{1/2}} = \frac{2\widetilde{H}}{\kappa} + y, \tag{3}$$

with boundary conditions

$$y' = -\cot\theta_1 \quad \text{at} \quad x = \kappa R, \tag{4a}$$

$$y' = 0 \quad \text{at} \quad x = \kappa L \quad \text{and} \quad y = 0. \tag{4b}$$

As pointed out in Ref. [22], Eq. (3) is invariant under the transformation $y \to -y$, $\theta_1 \to \pi - \theta_1$, and $\widetilde{H} \to -\widetilde{H}$, indicating the symmetry between a rising and a depressing meniscus. This second-order nonlinear differential equation can be rewritten in terms of the local tilt angle of the liquid-vapor interface, ϕ, as defined in Fig. 1. Since $y' \equiv \frac{dy}{dx} = \frac{dz}{dr} = \tan\phi$, Eq. (3) then becomes

$$\frac{d\sin\phi}{dx} + \frac{\sin\phi}{x} = -\frac{2\widetilde{H}}{\kappa} - y. \tag{5}$$