



# Development of wavelet deconvolution technique for impact force reconstruction: Application to reconstruction of impact force acting on a load-cell



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## ABSTRACT

Deconvolution technique has become a useful solution for indirect measurement of impact force, however, a simple deconvolution technique often encounters difficulty and leads to an unsatisfactory reconstruction due to unavoidable error and the ill-posed nature of the inverse problem. This paper is concerned with a deconvolution technique using wavelet approach for reconstructing impact force. Based on the ability of wavelet transform, impact force reconstruction was formulated and implemented through its expansion coefficients of different scaled and shifted versions of wavelets. The reconstruction of impact force thus becomes the reconstruction of its expansion coefficients which are then used to reproduce the profile of impact force. Considerations of appropriate scales and shifts of wavelets in reconstruction can be regarded as a kind of regularization in order to improve the ill-posed problem. The numerical verification on reconstruction of impact force acting on a load-cell during the impact buckling of thin-walled column showed that the present technique is indeed capable to mitigate the ill-posed nature when the scale level of wavelets is controlled properly. The result also revealed that with an appropriate choice of the shifting parameter at a certain scale level, the noise in reconstructed force has been suppressed further. As a consequence, it has been verified that the present wavelet deconvolution technique can successfully reconstruct the impact force with less noise and higher accuracy in comparison with the conventional methods.

## 1. Introduction

Understanding of impact force history is crucial in impact engineering. However, it is often the case that direct measurement of impact force is difficult or infeasible in practice. Taking this matter into account, indirect measurement or reconstruction of impact force from corresponding impact response such as strain, displacement or acceleration etc. has been widely considered so far. Inoue et al. [1] and Sanchez and Benaroya [2] thoroughly reviewed and summarized many studies on impact force reconstruction. The most common approach is deconvolution, that is to solve the linear convolution integral for the input (impact force) when the output (corresponding response) is given. There are mainly two ways for deconvolution: one is direct discretization of the convolution integral in the time domain and the other is the use of Fourier or Laplace transform in the frequency domain.

Unfortunately, deconvolution is often ill-conditioned and leads to an unacceptable reconstruction of impact force, especially in the presence of measurement errors that are unavoidable in reality. In order to

cope with this difficulty, many techniques have been applied in the literature as follows:

- (1) Least squares method using longer response data than the impact force data [3],
- (2) least squares method using multiple responses [4],
- (3) good selection of response location [4,5],
- (4) using mutual deconvolution [4],
- (5) non-negative constraint to the impact force [4], and
- (6) regularization technique such as the truncated singular value decomposition (TSVD) [6], Tikhonov method [7] or Wiener filter [8], etc.

Note that these references [3–8] can be considered to be original, to the authors' best knowledge, in spite of the fact that there are so many publications which also deploy such techniques.

Although these techniques are affirmed to be effective, a more robust technique is demanded. Application of wavelets has been

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concerned and examined because of the development of wavelet transform theory. By taking the wavelet transform, a signal function of an arbitrary impact force is represented as a linear combination of wavelets which are often compactly supported (finite in time). This is in contrast to the Fourier transform, in which the impact force is represented by a linear combination of sinusoidal functions (infinite in time). From this point of view, the use of wavelet approach for impact force reconstruction seems to be preferable since most impact forces have a finite duration in reality.

In early works by Doyle [9,10], wavelets were utilized to identify impact forces acting on a beam and a plate but only by considering shifting in the time domain. According to the fundamentals of wavelet theory [11], however, it is advantageous to utilize wavelets obtained not only by shifting but also by scaling of a basic wavelet. Inoue et al. [12,13] demonstrated the effectiveness of Gabor or Haar wavelet in reconstructing impact force struck on a rod, but it was just a basic verification with a simple model. Lifschitz and D'Attellis [14] applied wavelets for reconstructing the harmonic and impulsive force but there was no mention of the effects of scaling and shifting of wavelets on reconstructed results. Li et al. [15] also studied multi-resolution analysis of wavelets to reconstruct not impact force but periodic dynamic force. These authors used the filtering of high-frequency (small-scales) components of the force and response signals in order to reduce noise sensibility in deconvolution process. However, it may lead to inaccurate estimation due to the loss of high-frequency information. Furthermore, reconstruction in both references [14,15] was proposed in terms of the impulse response function (IRF) known in advance. This is actually another difficulty since the determination of IRF is also an inverse problem. Thus, the IRF may be unstable or erroneous due to the ill-conditioned nature of the inverse problem, and it can lead to an inaccurate reconstruction of impact force. In the similar manner as the use of wavelets, Gunawan et al. [16] utilized cubic B-spline basis to seek a more accurate reconstruction of impact force. Nevertheless, no scaling and shifting information of cubic B-spline function were expressed, and it is still difficult in determining the number of these basis functions. Qiao et al. [17] also used cubic B-spline scaling function for force identification based on the algorithm of wavelet multi-resolution analysis, but only the influence of scaling level on the estimated force was examined. More recently, Tran and Inoue [18] have investigated and exhibited the reconstruction of impact force in terms of considering both scaling and shifting of Haar wavelet.

However, reconstruction of impact force with the wavelet approach is relatively immature and there are few publications until now. Furthermore, as far as the authors know, there has been no study which evaluates the effects of both scaling and shifting of wavelets on impact force reconstruction. Therefore, the goal of the present work is to develop a comprehensive deconvolution technique using wavelets as a robust technique in order to reconstruct impact force history accurately by utilizing both scaling and shifting features of wavelets.

This paper is structured as follows. Next, Section 2 will briefly introduce the basic knowledge of impact force reconstruction by means of deconvolution. Section 3 proposes and formulates the reconstruction technique of impact force with the use of wavelets in the cases when IRF is determined explicitly and implicitly. After that, in Section 4, the technique is applied to a numerical experiment on reconstruction of different impact forces acting on a load-cell during the impact buckling of thin-walled column. Conclusions will be figured out in Section 5.

## 2. Basic scheme for impact force reconstruction by deconvolution

When an arbitrary structure is subjected to an impact force  $f(t)$ , if the induced response  $e(t)$  at a particular position on this structure linearly depends on the force, the relationship between this force and response can be represented by the following convolution integral

$$e(t) = \int_0^t h(t - \tau)f(\tau)d\tau = h(t)*f(t), \tag{1}$$

where  $h(t)$  is the impulse response function (IRF) which characterizes impact behavior of the system and  $*$  denotes convolution integral. It is assumed that  $h(t) = f(t) = 0$  for  $t < 0$  without loss of generality. By considering a finite signal with the time  $t$ , normalizing the time  $t$  by a constant time  $t_0$  as  $x = t/t_0$  and discretizing the normalized time  $x$  with a constant period  $\Delta x$  as  $k = x/\Delta x$  (namely,  $k = 0, 1, \dots, K - 1$  with  $K$  is the number of discrete points), the normalized and discrete version of Eq. (1) can be expressed as an algebraic equation

$$\{e\} = [h]\{f\}, \tag{2}$$

of which vectors  $\{e\}$ ,  $\{f\}$ , and the impulse response matrix  $[h]$  are

$$\{e\} = \{e(0) \ e(1) \ e(2) \ \dots \ e(K - 1)\}^T, \tag{3}$$

$$\{f\} = \{f(0) \ f(1) \ f(2) \ \dots \ f(K - 1)\}^T, \tag{4}$$

$$[h] = \begin{bmatrix} h(0) & 0 & \dots & 0 & 0 \\ h(1) & h(0) & \dots & 0 & 0 \\ h(2) & h(1) & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h(K - 2) & h(K - 3) & \dots & h(0) & 0 \\ h(K - 1) & h(K - 2) & \dots & h(1) & h(0) \end{bmatrix}, \tag{5}$$

respectively.

## 3. Impact force reconstruction with wavelet deconvolution technique

### 3.1. Wavelet expansion of impact force

According to the theory of discrete wavelet transform [11], the impact force  $f(x)$  can be approximately represented as

$$f(x) = \sum_{m=0}^M \sum_{n=n_0}^{N_m} \tilde{f}_{m,n}^d \psi_{m,n}(x) + \sum_{n=n_0}^{N_M} \tilde{f}_{M,n}^a \varphi_{M,n}(x). \tag{6}$$

Here,  $\psi_{m,n}(x) = a_0^{-m/2} \psi[(x - nb_0 a_0^m)/a_0^m]$  and  $\varphi_{M,n}(x) = a_0^{-M/2} \varphi[(x - nb_0 a_0^M)/a_0^M]$  are scaled and shifted versions of the wavelet function  $\psi(x)$  and the scaling function  $\varphi(x)$ , respectively, in which  $a_0^{-m/2}$  is a normalization constant,  $a_0^{-m}$  and  $nb_0 a_0^m$  represent the scaling and shifting parameters, respectively;  $m$  ( $m_0 \leq m \leq M$ ) and  $n$  ( $n_0 \leq n \leq N_m$ ) are integers. In addition,  $\tilde{f}_{m,n}^d$  and  $\tilde{f}_{M,n}^a$  are respectively the expansion coefficients at a certain scaling level  $m$  and  $M$ ; the superscripts  $d$  and  $a$  denote the detail and approximation terms, respectively.

### 3.2. Wavelet analysis for impact force reconstruction

#### 3.2.1. Using the impulse response function explicitly

Suppose that the IRF of an impact system in Eq. (1) is given a priori. Substituting Eq. (6) into Eq. (1) and using the normalized time  $x$ , then the relationship between impact force and corresponding impact response will be represented by the following summation

$$e(x) = \sum_{m=0}^M \sum_{n=n_0}^{N_m} \tilde{f}_{m,n}^d \phi_{m,n}(x) + \sum_{n=n_0}^{N_M} \tilde{f}_{M,n}^a \chi_{M,n}(x), \tag{7}$$

where

$$\phi_{m,n}(x) = \int_0^x h(x - \xi) \psi_{m,n}(\xi) d\xi = h(x) * \psi_{m,n}(x), \tag{8}$$

$$\chi_{M,n}(x) = \int_0^x h(x - \xi) \varphi_{M,n}(\xi) d\xi = h(x) * \varphi_{M,n}(x) \tag{9}$$

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