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An analytical investigation on large post-buckling behavior of a drilling shaft modeled as a rotating beam with various boundary conditions

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ABSTRACT

Post-buckling behavior of a drilling shaft modeled as a rotating beam subjected to terminal force is investigated in this paper. Three boundary conditions: clamped-clamped, clamped-free, and clamped-hinged are considered. Classical Bernoulli-Euler beam model including shear force is applied to depict the problem. In particular, for clamped-hinged ends boundary condition, the analytical approximate solutions are more difficult to construct, due to asymmetric boundary and configuration. Unlike numerical results in the existing literatures, analytical approximate solutions of the problem above are established via combining the Maclaurin series expansion, orthogonal Chebyshev polynomials, Galerkin and Harmonic balance method in this paper. Compared with numerical solutions obtained by using the shooting method on the exact governing equation, the approximate analytical solutions here show excellent accuracy and rapid convergence. The effect of the system parameters on the dynamic response and stability of system can conveniently be investigated via the present analytical approximate solutions. In conclusion, the analytical approximate solutions presented here are sufficiently precise for a wide range of the maximum angle of the rotating beam.

1. Introduction

A rotating beam can be used to model turbine shafts, rotor systems, spinning missiles, deep well drill pipes, micro-drill system and other mechanical elements [1–3]. It is significant for the design of shafts, rotating machinery and drilling engineering devices to predict buckling or whirling vibration of an axially rotating beam [4]. Behzad and Bastami [5] studied the dynamic problem of a rotating uniform cylindrical shaft with an Euler-Bernoulli beam model. Three boundary conditions including pinned-pinned, pinned-clamped and clamped-clamped were considered. They found that centrifugal force did play an important role on natural frequency of a rotating shaft. Recently, Timoshenko beam theory was used to investigate the rotating shaft with linear and uniform circular section and various ends [6]. It is concluded that the present Timoshenko model is more accurate than Euler-Bernoulli one. Furthermore, the natural frequencies could be overestimated in Euler-Bernoulli and Rayleigh models [7,8]. As the spiral groove on the surface of a micro-drill is shallow and narrow, the center of the drill is thick. A micro-drill fixed in drill chuck can be regarded as a cantilever cylinder [9]. Micro-drill-spindle system dynamic behavior was investigated by Pei, et al. [10]. A dynamic model with the beam of circular cross section was established and several factors on dynamic stresses of micro-drills were proposed. In order to simulate the actual situations that a micro-drill did not contact, just contacted, and penetrated completely into the work-piece, Cheong et al. [11] presented three boundary conditions, namely, clamped-free, clamped-pinned and clamped-clamped. Based on elliptic integrals, the re-expression of the equilibrium equation of beams with different supported ends was provided by Humer [12] and Prechtl et al. [13]. The post-buckling shapes were obtained, the accurate closed-form solutions of the axial displacement were also derived. Gupta et al. [14] derived the rigid closed-form expressions for the post-buckling deformation of a composite beam with common boundary conditions with the Rayleigh-Ritz method. However, the researches mentioned-above are based on static model, the shear force is always a constant.

The rotating beam exhibits complicated motion. The rotating slender beam with a constant angular velocity tends to buckle. Shear force is not a constant any more under the action of centrifugal forces, which will increase great difficulty for giving closed form solution. Bobisud and Christenson [15] studied the stability of a rotating rod with an axial force and clamped-hinged boundary condition. The research has practical engineering significance in the stability of drill bits. The post-buckling of a rotating compressed rod with the cantilevered support

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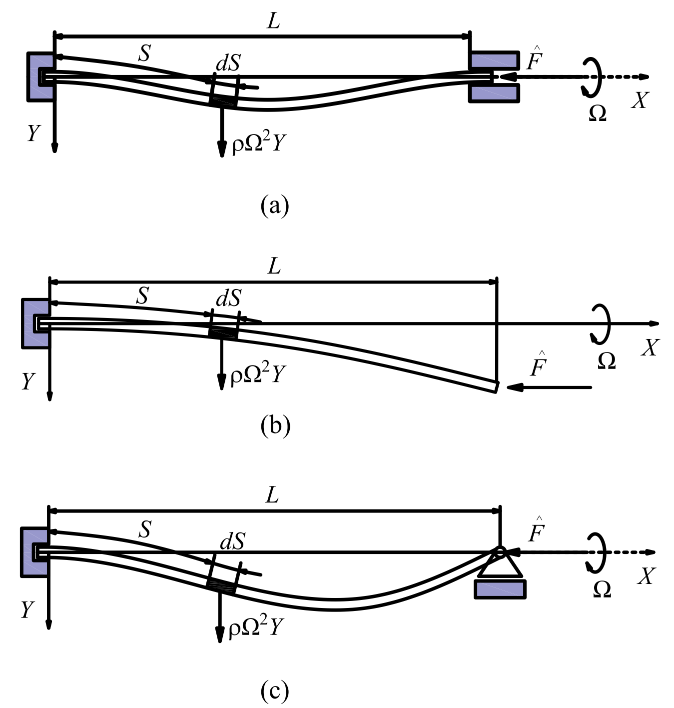

Fig. 1. Rotating slender elastic beams with different boundary conditions: (a) clamped-clamped, (b) clamped-free, (c) clamped-hinged.

condition was constructed by Atanackovic [16]. Based on the extremum variational principle, the equilibrium equations of the rod with an axial load were constructed and then approximate analytical solutions were derived. Owing to the asymmetric boundary conditions, the influence of shear stresses must be concerned. In 2000, a rotating cantilevered and finitely deformed elastic rod was surveyed by Atanackovic [17] again with the variational principle and linearized perturbation equations. Bodnar [18] concerned the bifurcation solutions of a rotating rod with hinged ends, while Wang [19] focused on more complex boundary conditions (fixed-hinged). On the other hand, many scholars have devoted themselves to the research of the optimal shape of a rotating slender rod. The optimal shapes of a compressed rotating rod were proposed by Atanackovic [20,21] and Braun [22]. By employing Pontryagin's maximum principle, the determination of the optimal cross sectional area function was simplified as the solution of the nonlinear boundary value problem. This shape was not the same as the result via the Bernoulli-Euler theory.

Recently, the nonlinear problem of large post-buckling of a rotary compressed nanorod with fixed-free ends were discussed by Atanackovic and Zorica [23]. Critical values of angular speed and compressive force were determined. Post-buckling shape of the rod was also obtained by calculating equilibrium equations. Using similar constitutive relations, the post-buckling deformation of a rotating compressed nanorod with fixed-fixed ends were studied by Atanackovic et al. [24]. However, the analytical solution for the post-buckling of a rotating slender compressed rod with clamped-hinged ends has rarely been studied.

This paper is concerned with the post-buckling problem of a drilling shaft modeled as a rotating beam with fixed-fixed, fixed-free and fixed-hinged ends. An effort is presented to extend and generalize the previous analytical approximate analyses to nonlinear dynamical systems [25–27]. The configurations of the post-buckling are depicted in Fig. 1. Owing to the statically indeterminate boundary conditions and the centrifugal force from rotating, the shear force cannot be ignored, and it is not a constant value. The equilibrium equations are derived in the nonlinear form contained with both sine function and cosine function. Employing the Maclaurin series expansion and orthogonal Chebyshev

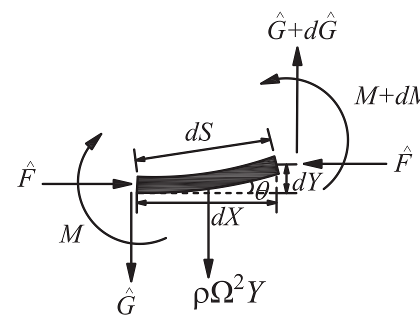

Fig. 2. A force analytical sketch of a rotating slender elastic beam.

polynomials, the governing equations are rearranged. The trigonometric functions in the governing equations are reduced to polynomial expression. Therefore, the analytical expressions of the large deformation could be proposed in the functions of the maximum angle of the rotating beam. In addition, the solving processes of analytical approximate solutions for post-buckling of the rotating beam yield rapid convergence to numerical solutions obtained via the shooting method. The present results are valid for a large range of the rotating beam angle amplitude.

2. Mathematical model and solution methodology

Consider an elastic beam rotating with angular velocity Ω and axial compressive force $\hat{F}$, where the arc length ordinate of the beam is $S \in [0, L]$. In this investigation, only steady state of a rotating shaft or drilling string is concerned with. Therefore, the angular velocity Ω is assumed to be constant in the model, namely, the angular acceleration is zero. A force analysis schematic of the rotating slender elastic beam is displayed in Fig. 2.

Based on the balance of forces and moments and geometrical relationships, the governing equations of a drilling shaft modeled as a rotating beam are derived. The non-dimensional form are shown [19]

$$\frac{d^2\theta}{ds^2} + F\sin\theta + G\cos\theta = 0, \tag{1}$$

$$\frac{dG}{ds} + J^4 y = 0, \tag{2}$$

$$\frac{dy}{ds} = \sin\theta, \tag{3}$$

$$\frac{dx}{ds} = \cos\theta, \tag{4}$$

where

$$F = \frac{\hat{F}L^2}{EI}, G = \frac{\hat{G}L^2}{EI}, y = \frac{Y}{L}, x = \frac{X}{L}, s = \frac{S}{L}, J^4 = \frac{\rho\Omega^2 L^4}{EI}. \tag{5}$$

Here (X, Y) constitutes Cartesian coordinates of the beam, $\hat{G}$ is the shear force, ρ is the mass density of the beam, L is the length of the beam, and EI is the flexural rigidity of the beam, respectively.

2.1. Clamped-clamped rotating beam

The post-buckling of a beam with both ends fixed is firstly discussed. In this case, the post-buckled configuration is symmetric, the maximum deflection is at the midpoint of the structure $s = 1/2$. The maximum of the angle between deformed beam and undeformed configuration appears at $s = 1/4$. The non-dimensional boundary and geometry conditions are

$$\theta(0) = 0, \quad \theta(1/4) = a, \quad \theta(1) = 0, \quad y(0) = y(1) = 0, \quad x(0) = 0. \tag{6}$$

Using Eqs. (2) and (6), the boundary conditions of shear force G are shown

$$\frac{dG}{ds}(0) = 0, \quad \frac{dG}{ds}(1) = 0. \tag{7}$$