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Research paper

An innovative determination approach to tooth compliance for spiral bevel and hypoid gears by using double-curved shell model and Rayleigh–Ritz approach

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ABSTRACT

In high-performance sophisticated flank design for the spiral bevel and hypoid gears, tooth compliance is always an important input for identifying the tooth mechanical properties. Distinguished with the conventional modeling, a new accurate finite element structure model is established to determine tooth compliance by using elasticity-based deformation solution. In full consideration of the flank flexural behavior characteristic, a double-curved shell model with varying thickness is established to get a more accurate representation of geometric shape for face-milling or face-hobbing gear than a beam or plate model. In determination of tooth compliance, the high order shear theory of Bhimaraddi shell is used to set the displacement assumption, and the Rayleigh–Ritz approach having algebraic polynomial trail functions is performed to obtain an expression for the transverse deflection and shear rotation by considering the geometric boundary conditions. A given numerical result for the face-milling spiral bevel and hypoid gear is provided to verify the proposed methodology by comparing with the loaded tooth contact analysis (LTCA) using well-known finite element method (FEM).

1. Introduction

Spiral bevel and hypoid gears as the mechanical structure with complex geometric and properties, are widely used to transfer mechanical power between the intersecting axes in automotive, aircraft and ship industries. More recently, the collaborative tooth flank design considering both geometric and physical performances [1,2] is always an active point. The less noise and more strength has been a major concern in gear transmission system, as not only the environmental requirements but also customer demands must be met. Especially for the gear strength design including tooth contact stress and root bending stress in case of load, the accurate numerical solution method is very complicated and challenging, due to complex geometric flexural behavior and distinctive contact form [3]. At present, in the design of gear loaded strength, the most commonly used technique is simulation analysis based on finite element method (FEM) [4]. Where, the loaded tooth contact analysis (LTCA) based on finite element simulation software [5], as an important mean of simulating tooth contact state, is always employed to predict and evaluate gear strength. However, there are some deficiencies in this simulation.

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(i) It focuses on synthesis and analysis for quasi-static mechanical properties, and lacks of accurate solution for dynamic mechanical properties which is of more importance to gear transmission [6].
(ii) The computation for LTCA is complex and time-consuming. It generally needs to take days CPU times to solve for the relative evaluation items by 2.5 GHz processor and 1 Gbyte RAM [7].
(iii) This simulation lacks of flexibility and versatility. It is performed based on encapsulated commercial software package [8] and may hardly be modified and processed immediately. Furthermore, once the results have problems, we can only modify the geometric solid model. However, the provided model is generally fixed, and its modification is usually miscellaneous and toilsome [9].

It is well known that the tooth compliance is the basic input to solve for gear dynamics mechanical properties [10]. With the calculated tooth compliance, the governing equation for gear dynamics analysis can be established to solve for some evaluation items [11], such as vibration frequency and amplitude, can be obtained. Moreover, as mentioned in Ref[12]. by Kahraman, the determination of tooth compliance forms the basis for almost all prediction of required items in LTCA, including the load distribution, loaded contact pattern, contact stress and load transmission error.

1.1. Literature review

As for computing the gear tooth compliance, there are two main methods, namely theoretical and numerical ones [11]. In theoretical calculations, there are the elastic mechanics and material mechanics method. The former is the conformal mapping method using complex function of mathematics and elasticity. And the latter is the calculation formula of tooth compliance based on classical material mechanics theory, where the deformation is firstly derived and submitted into it. The typical calculation formulas include Ishikawa formula and Weber formula. In the numerical calculations, there are three main types: FEM, boundary element method, regression method. The FEM plays an important role in solving the complex physics problems with the arbitrary geometry problem [12]. It generally follows the following calculation steps. It firstly establishes the gear three-dimensional solid model. And then, the finite element theory is employed to analysis the established model and to obtain the contact force deformation. Finally, it needs to deal with the contact force and the deformation considering the geometric boundary conditions, and the tooth compliance at a certain contact position is identified.

In the most decades, however, tooth compliance is mainly researched on the straight gear or helical gear and rarely on the spiral bevel and hypoid gear [11,12]. As for the straight gear, the most frequently used method is Ishikawa formula or Weber formula [13]. He [14] determined the tooth compliance using FEM for spur and helical Gear. As for the helical gear, Lin [15] and Simmon [16] proposed the estimation formula. Where, the statistical experiment method is used to make regression analysis of large number of result data of finite element calculation. Gosselin [17] developed the tooth compliance by using the Finite Strip Method. However, it has only analyzed the deformation when the load was applied at a certain point and not studied on compliance variation in a meshing cycle. Consequently, it cannot obtain the compliance curve for providing dynamic analysis of gear system. As for the spiral bevel and hypoid gears, LTCA based on finite element simulation is a major means to numerical solution. The geometry of tooth flank for spiral bevel gear is so complex that it usually needs to be simplified to some extent in theoretical or numerical calculation. Though it is very difficult to perform accurately the mechanics analysis in case of load, the mechanical properties of spiral bevel gear is calculated by FEM is of obvious advantage. Simon [18,19] calculated the loaded deflection and displacement using regression analysis of finite element results. Litvin [20,21] made synthesis and analysis of finite element hypoid gears to obtain high contact performance design. Subsequently, most designers were keen on this simulation design using software which is similar to the black box [22]. Whereas, in this simulation, the key point to computing the mechanical properties of the tooth surface, namely tooth compliance is ignored. Meanwhile, the detailed representation and establishment of finite element model of tooth compliance is sparsely researched. Gosselin et al. [23] proposed a curve-fitting method to the finite element deformation results for calculating tooth compliance. Wilcoxet et al. [24] provided a three-dimensional model of gear tooth including base deformations to identify the tooth compliance.

However, most gear designer usually ignored the establishments of finite element mode in determination of tooth compliance. To establish finite element model, the complex geometry of spiral bevel and hypoid gears is usually simplified. A semi-analytical model is usually established by using elasticity-based deformation. In these established models, the cantilever beam is firstly used to represent finite element unit. Gosselin used the cantilever beam of Westinghouse to estimate the tooth compliance [25,26]. Then, it adopted the plate model. Yakubek et al. [27] used a tapered plate model to estimate the compliance of spur and helical gears. Here, the linear thickness variation along profile direction is considered, which is different with the Timoshenko plate solution [28]. Yau et al. [29] provided shear tapered plate model to determine bending and shear deflection of gear teeth. Where, tooth bending deflections is taken account a sum of shape functions satisfying the given boundary conditions, and the unknown coefficients of the shape functions were determined by mining the potential energy. Vaidyanathan et al. [31] developed an analytical tooth compliance model using Rayleigh-Ritz method. Subsequently, shear deformation and based rotations [30] were considered to identify the tooth compliance. In the consideration of accurate gear geometric topography, he paid attention to the linearly varying thickness in the profile and lengthwise directions as well as linearly varying tooth height along the face width direction. It is a pity that above analytical compliance methods were valid for gear teeth having constant height along its face width direction and either constant or linearly varying thickness along its profile, which is not suitable for spiral bevel and hypoid gears. Thence, the complex shell theory was applied