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Research paper

Form-closure of planar mechanisms: Synthesis of number, configuration, and geometry of point contacts

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ABSTRACT

This paper presents a study on restraining the motion of planar mechanisms using point contacts between the links of the mechanisms. First-order form-closure (FFC) and second-order form-closure (SFC) of 1- and 2-degrees-of-freedom (d.o.f.) smooth planar motions are studied. It is shown geometrically that two and three point contacts are sufficient for FFC of an object which is capable of executing 1- and 2-d.o.f. motions respectively. These results are applied for FFC of planar mechanisms. A theory for contact curvature based SFC of 1- and 2-d.o.f. motions with a single point contact is presented. It is derived that the instantaneous point-path curvature of points on a specific line of a 2-d.o.f. moving plane is always bounded. Using this result, a procedure is chalked out for immobilizing 1- and 2-d.o.f. mechanisms with a single contact. Illustrations of immobilizing four-bar and 2R-mechanisms are presented. Rapid prototyped model of the four-bar mechanism and stoppers is fabricated and the SFC theory is verified.

1. Introduction

A rigid body executing a planar motion has at most 3-d.o.f. The problem of immobilizing an unconstrained free rigid body which has 3-d.o.f. with point contacts is well reported in the literature on form-closure [1–5]. Form-closure of multiple connected bodies constituting a mechanism, is of practical utility where the mechanism need be locked when not in use or at a particular configuration for a definite purpose. Such an immobilization is required in diverse applications of deployable systems. For example, once an antenna is deployed, it should not further open up or close-in. Rigidization of such deployable structures and inflatable mechanisms is an important area where immobilization finds its way [6]. Most of the present applications use unidirectional stoppers wherein multiple stoppers are required for restraining the motion of the mechanism. The use of multiple point contacts results in uncertain performance.

Form-closure of a system of connected rigid bodies capable of articulation, mechanisms in particular, is a relatively less explored area. Using screw systems, Dizioğlu and Lakshminarayana [3] analytically determined the number of wrenches required for force-closure of rigid body with less than 6-d.o.f. in space and less than 3-d.o.f. in plane. Rimon and Stappen [7] presented form-closure of *free planar connected serial polygons* with *pointed fingers*. The second-order theory developed there is specific to the special case of free connected polygons in contact with point fingers since the analysis is based on a special *circle property* first reported in [8]. The contact geometry is limited to rectilinear edges of bodies in contact with

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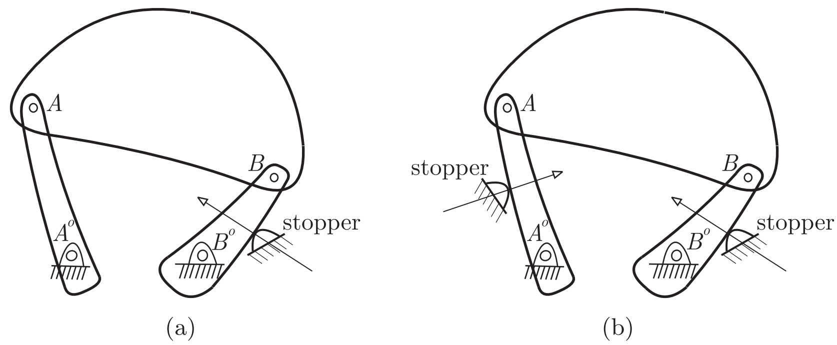

Fig. 1. Restraining motion of four-bar mechanism: (a) one way blocked with single stopper, (b) completely blocked with two stoppers (FFC).

point fingers and limited to serial kinematic chains. Cable supported platforms and mechanisms is an emerging area of research. Mustafa and Agarwal [9] studied force-closure of cable driven n-d.o.f. serial chains and showed that $n+1$ cables with positive tension are sufficient to constrain such chains. Form-closure of general closed loop mechanisms is not dealt with in the existing literature. From the applications in mechanical engineering point of view, *form-closure of mechanisms is more pertinent* and is not well addressed by the previous researchers.

In the present work, the concept of form-closure is extended to the immobilization of planar mechanisms. In certain mechanisms like those in electrical switches, stoppers are generally used to restrict the range of motion of the mechanism [10,11]. When a link of the mechanism is in a point contact with a stopper, it prevents all the possible motions of the link for which the point of the link in contact with the stopper has a velocity component penetrating into the stopper. If multiple such stoppers are used, a mechanism can be immobilized. This is termed as *first-order form-closure of a mechanism in the present work*. More precisely, *a mechanism is said to be in FFC if every first-order instantaneous motion of it is blocked by the contacts*. FFC of a mechanism requires an arrangement of stoppers such that the mechanism cannot move. For example, in Fig. 1, two stoppers each one in contact with the two pivoted links arrest the motion of the four-bar mechanism. The curvatures of the link and stopper at the point of contact does not matter here; only the contact normal information is required. The idea of second-order form-closure (SFC) is also very much applicable to mechanisms. SFC of mechanisms is defined here as follows. *A mechanism is said to be in SFC, if every second-order motion of it is blocked by the contacts and not every first-order motion is blocked by them*.

In this paper, FFC and SFC of 1- and 2-d.o.f. smooth planar motions are studied. Section 2 introduces the essential fundamental concepts of contact geometry, motion and form-closure. The relation between motion and contact geometry is discussed in Section 3. In Section 4, it is shown that two and three point contacts are sufficient for FFC of an object which is capable of executing 1- and 2-d.o.f. motions respectively. The number and also the location of the contacts for FFC of 1- and 2-d.o.f. planar mechanisms are derived using geometric reasoning in Section 5. A theory for curvature based SFC of 1- and 2-d.o.f. motions with a single point contact is presented in Section 6. In Section 7, it is established that, through proper selection of curvatures at the contact, SFC of planar mechanisms can be achieved with a single point contact. The theory is validated through a rapid prototyped model. *According to the state of the art, a stopper restrains only uni-directional motion of a link since the analysis is only up to first-order instantaneous kinematics. But, we show that a single stopper can inhibit bi-directional motion based upon the geometric characteristics of relative motion up to second-order instantaneous kinematics*. It is derived that the instantaneous point-path curvature of points on a specific line of a 2-d.o.f. moving plane is always bounded. This result paved the way for SFC of 2-d.o.f. mechanisms dealt in Section 7.

2. Fundamental concepts

The basic concepts, available in standard literature, relating to the geometry of contacting curves, relative motion between them have been paraphrased in this section for the sake of completeness and appropriate comprehension of the subsequent sections.

2.1. Contact between curves

Material side of a curve: An oriented simple curve can define a neighbourhood of points which are locally and persistently lying either to the left or to the right of the curve within an arbitrarily small region of interest; one of the two sides, either right or left, can be defined as the material side for which the curve is the boundary.

Intersecting curves: If for no definition of material side of two curves at a given region of interest, the material sides are disjoint, then the curves are intersecting.

Contact between curves: Two curves are said to be in contact if they have point(s) in common but their material sides are disjoint for some definition of the material side; the common points are referred to as points of contact.