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Research paper

On the stiffness analysis of robotic manipulators and calculation of stiffness indices

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ABSTRACT

This paper aims at introducing new performance indices for robotic manipulators in order to evaluate the robot stiffness at the design embodiment stage. In this regard, the calculation of the Cartesian stiffness matrix of a manipulator is elaborated based on a matrix structural analysis methodology. Then, by resorting to linear algebra, four stiffness indices, two for translational and two for rotational deformation of the end-effector, are extracted from a Cartesian stiffness matrix. It is proved that the indices represent the maximum and the minimum value of the resistance forces or moments of a manipulator against an exerted deflection, on the end-effector. As a case study, the foregoing stiffness analysis will be applied on a Delta parallel robot and the corresponding stiffness indices will be derived.

1. Introduction

At the design embodiment level, the stiffness analysis of parallel manipulators is an essential task. In industrial pick-and-place operations, high-speed robot manufacturers tend to decrease moving masses to achieve high acceleration and reduce the operation time. However, this affects the manipulator resistance against the external forces. In robotic based machining, grinding, trimming etc., the manipulator is subjected to significant external forces which may lead to large deflections of the end-effector. In medical robotics, for example, elastic deformations due to the payload or the links weight can be very harmful. Moreover, stiffness analysis is also important in the design of accelerometers using microelectromechanical systems. In such cases, the structure is designed to provide flexibility along a desired direction and high stiffness along the remained directions [1,2].

In general, stiffness modeling of manipulators has been mainly conducted by three approaches: 1) Finite Element Analysis (FEA), 2) Virtual Joint Method (VJM) and 3) Matrix Structural Analysis (MSA). FEA is a common method for stiffness analysis in which the physical model is divided into a number of finite elements and nodes. Then, the compliant relations between the adjacent nodes are derived and solved simultaneously. Two advantages of FEA in robotic applications are its high accuracy and the capability of links and joints modeling with actual shapes. However, FEA is expensive in terms of CPU time. Although higher number of elements leads to a better accuracy, the cost of computations is a limitation. In this regard, Corradini et al. analyzed the stiffness of the H4 parallel robot by FEA at one single posture, and the results have been verified experimentally [3].

The VJM approach was first introduced by Salisbury [4], and then it is elaborated by Gosselin [5]. In this method, the links are considered rigid but the joints are assumed to be flexible. This lumped presentation of the manipulator stiffness,

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adds some auxiliary virtual to the kinematic pairs with embedded virtual springs. One of the challenges of this method is how to define the virtual spring parameters [6]. First, the main sources of elasticity were assumed to be concentrated in the actuated joints and the corresponding elasticity was modeled with one-dimensional virtual springs [7–9]. Then, the flexibility of the links are taken into account either by including additional translational and rotational virtual springs located at the joints [10] or using 6-dimensional virtual joints representing the flexibility of each link [11–13]. Later, the VJM approach was significantly improved by Pashkevich and his colleagues [14–17]. Recently, FEA was incorporated in order to calculate the virtual spring stiffness values [18]. Deriving the CSM in VJM, is based on deriving the force-balance equations of the system using the principle of virtual work. Klimchik et al. used VJM for the evaluation and identification of industrial manipulators' stiffness parameters [19]. This novelty increased the accuracy of VJM. Although the VJM was first introduced for serial manipulators, it was efficiently implemented on parallel robots as well [14,20]. However, the VJM is related to some difficulty in stiffness modeling of complex geometries such as parallel manipulators, e.g., for architectures with internal loops (or with parallelogram links), the VJM is moderately complicated [6].

The MSA method is similar to FEA, while, instead of dividing the physical model into a large number of elements, each part of the robot (e.g. link, joint and actuator) is treated as a simple structural element (e.g. beam and rod) and the stiffness matrix of the whole structure is obtained by calculating the Hessian of elastic potential energy of the whole system. In MSA, displacements of each node (manipulator's active or passive joints) has a physical interpretation, which can be useful in some applications [21,22]. The main advantages of the MSA are reducing the computational expenses (compared to FEA) and the ability of obtaining the stiffness matrix of the structure in parametric form; this is crucial when an optimization of the stiffness behavior is demanded. Delbaise et al. computed the Cartesian stiffness matrix of the Delta parallel robot by using the classical MSA method [23]; also, GonÃ§alves et al. employed the MSA for the stiffness analysis of the 6-RSS robot [24]. Cammarata and Sinatra applied the MSA method to spherical parallel machines with curved links incorporating Curved Timoshenko beam element [25]. Cammarata has also developed an extended MSA method to include preload, external wrenches, flexible passive and actuated joints and deformable links inside a unified mathematical formulation [26].

A "small"-amplitude displacement (SAD) screw is defined in the context of screw theory, wherein a six-dimensional array of Plücker coordinates, of which four are independent, represent a line [27]. A screw is a line array with a pitch as the fifth independent parameter. The amplitude *A*, multiplying the screw array, is the sixth independent parameter which defines a twist $\mathbf{t}$—point velocity and angular velocity—or a wrench $\mathbf{w}$—force and moment—depending on the unit of *A*. The SAD screw is defined as $\mathbf{u} = \mathbf{t}\Delta t$, where the product $|\boldsymbol{\omega}|\Delta t \ll 1$. Elastostatic analysis of a robot characterizes the manipulator resistance to a SAD screw caused by an external wrench applied on the Moving Platform (MP) [28]. Matrix $\mathbf{K}$ which maps the small amplitude SAD screw into the wrench applied on the MP is referred to as the Cartesian Stiffness Matrix (CSM). Recently, screw theory has attracted researchers for the design of compliant mechanisms [29–31]. Angeles investigated the nature of the CSM, using eigenvalue problem analysis by means of screw theory [1]; Taghvaeipour et al. introduced a formulation in the context of screw theory for the modeling of articulated flexible links [32]. In this regard, Zou and Angeles [2] conducted the stiffness analysis of a class of accelerometers by resorting to a generalized eigenvalue problem. Liu et al. [33] introduced a stiffness modeling approach of parallel mechanisms, by combining screw theory with the VJM.

If the components of the SAD screw are regarded as the generalized coordinates of a mechanical system [1], the Hessian matrix of the potential energy function with respect to the generalized coordinates yields the 6×6 stiffness matrix which is symmetric, positive definite or semi-definite. This matrix can be non-diagonal due to the coupling between the translational and rotational displacements [34].

With the foregoing methodologies, the 6×6 Cartesian stiffness matrix can be readily calculated, however, a scalar performance indicator is needed in order to clearly evaluate the stiffness of a robot at different postures, or to compare the stiffness of different robots at a single posture of a trajectory. Recently, different approaches were proposed by researchers; one set of possible candidates are the norm, determinant and trace of the CSM [5,35–37]. However, the 6×6 Cartesian stiffness matrix is composed of the 3×3 rotational, translational, and coupled stiffness blocks with different physical units [38], and hence, the CSM does not admit a norm, determinant or trace. Moreover, the coupling prevents an independent translational and rotational stiffness analysis. Thus, researchers conducted a generalized eigenvalue analysis [39–41] because of their physical interpretation. Researchers who conducted the eigenvalue analysis of CSM, believe that the eigenvectors provide the directions of maximum and minimum stiffness performance [36] and the eigenvalues can be used for drawing graphical representations of the stiffness behavior such as stiffness ellipsoids [5,41]. However, the CSM is not homogeneous in terms of units and hence, the eigenvalue analysis of CSM is not invariant with respects to those units. To deal with this problem, some researchers split the CSM into its equivalent translational and rotational parts [32,41]. Under certain conditions, Angeles discussed the decoupling of the Cartesian stiffness matrix in [1]. Later, Zou and Angeles introduced a decoupling technique of the Cartesian stiffness matrix by means of a similarity transformation that involves only a shift of the inertial coordinate origin [2]. However, the proposed technique is only applicable to those Cartesian stiffness matrices which have a singular 3×3 coupling block of rank 2 or 1 [1,2]. This singularity condition exists only for some particular types of rigid bodies (e.g. microaccelerometers) which have flexibility along some axes and rigidity along the rest. Patterson and Lipkin [42] suggested another graphical tool for a comparison of stiffness performance which is obtained through eigenscrew decomposition. Patterson and Lipkin described special cases in which compliant axes exist. The application of translational and rotational deformations coincident with a compliant axis leads to the decoupled rotational and translational behavior as investigated in [43]. Huang and Schimmels [44] decomposed the spatial stiffness matrix and introduced stiffness-coupling index using eigenscrew decomposition which can lead to a physical appreciation of compliance/stiffness