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Research paper

Robust balancing control of flexible inverted-pendulum systems

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ABSTRACT

This work studies the balancing control problem for flexible inverted-pendulum systems and investigates the relationship between system parameters and robustness to disturbances. To this end, a new energy-shaping controller with adaptive disturbance-compensation for a class of underactuated mechanical systems is presented. Additionally, a method for the identification of key system parameters that affect the robustness of the closed-loop system is outlined. The proposed approach is applied to the flexible pendulum-on-cart system and a simulation study is conducted to demonstrate its effectiveness. Finally, the control problem for a classical pendulum-on-cart system with elastic joint is discussed to highlight the similarities with its flexible-link counterpart.

1. Introduction

Flexible mechanical systems have been attracting the attention of the research community due to their light weight, smaller energy consumption and reduced manufacturing costs compared to traditional rigid structures [1]. Additionally, flexible mechanisms are particularly suitable for human-robot interactions and for operating in unstructured environments due to their intrinsic compliance [2,3]. Typically, flexibility can be either confined to unactuated joints connecting adjacent rigid links, or can be achieved using compliant links. In the latter case, advanced manufacturing technologies have been employed for the fabrication of flexible continuum robots [4]. From a modelling standpoint, additional unactuated degree-of-freedom (DOF) are generally associated to each compliant body employing a range of different methods. Notable approaches include: the pseudo-rigid-body-model (PRBM) [5,6] is widely used for kineto-static analysis; continuous models of flexible links for force and vibration control were proposed in [7–10]; a constrained Hamiltonian formulation [11] and a constrained Euler–Lagrange formulation [12] were recently employed with continuous models for the case of large deformations. In summary, flexible mechanisms are typically underactuated since they possess more DOF than actuators. Within this category, inverted-pendulum systems are particularly valuable from a theoretical standpoint and have been serving as models for wheeled robots [13], robotic fingers and manipulators [14], and foldable structures [15]. Additionally, the flexible inverted-pendulum represents an interesting starting point for the study of robot-assisted needle insertion in percutaneous interventions [16], which motivates this work. While extensive research has been conducted on the control of conventional inverted-pendulum systems, relatively few results exist for the flexible-link pendulum.

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Differently from [7–10], this work investigates the robust balancing control of systems consisting of an underactuated inverted-pendulum mounted on an actuated cart. Additionally, a continuous model of the flexible pendulum [12] is compared with the corresponding PRBM for the purpose of closed-loop control. The balancing control of inverted-pendulum systems typically aims to stabilise the upright position, which is unstable in open-loop. In this respect, optimal control [14], H-infinity control [17], and energy-shaping [18] are among the most notable approaches. In particular, energy-shaping control aims to appropriately define the total energy of the closed-loop system enforcing a strict minimum at the desired equilibrium and relies on either Euler–Lagrange or Hamiltonian formulation. In spite of its effectiveness, which has been demonstrated for several underactuated mechanical systems [19], energy-shaping control typically requires solving a set of partial-differential-equations (PDE) which can become problematic for larger and more complex systems. In order to circumvent this obstacle, different approaches have been devised: in [20] the closed-loop energy is chosen to be dependent on the control input warranting more flexibility in the definition of the PDE; in [21] the PDE are avoided altogether introducing the notion of algebraic solution. Nevertheless, both approaches result in a more elaborated controller design and in a relatively large number of parameters. Recently, an alternative energy-shaping design that obviates the solution of PDE while maintaining a more intuitive structure akin to a PID was proposed in [22–24] and applied to a flexible inverted-pendulum in [25], although external disturbances were disregarded for simplicity. While recent works proposed robustness enhancement strategies for energy-shaping control [26–28], most research has been focusing on matched disturbances (i.e. only affecting the actuated DOF) and still relies on the solution of PDE. In summary, the robust control of flexible inverted-pendulum systems with matched and unmatched disturbances (i.e. affecting actuated and unactuated DOF), which are common to many applications, remains an open problem of high practical relevance. Finally, research on design, modelling and control of flexible mechanisms has typically proceeded in parallel and with limited interactions between the different disciplines. Nevertheless, a concurrent design and control approach, which was proved effective for rigid systems [29], has been indicated as a promising avenue for flexible mechanisms [30].

The first contribution of this work is a new control scheme for a class of underactuated mechanical systems with constant disturbances. To this end, the energy-shaping control [22] is revisited and augmented with an adaptive disturbance-compensation term, while a rigorous stability analysis is conducted employing Lyapunov functions. As second contribution, the relationship between system design and robustness to disturbances is investigated, a method for the identification of key system parameters is presented and tuning guidelines are outlined. Finally, the control scheme is applied to the flexible pendulum-on-cart and to its rigid-link counterpart defined according to the PRBM paradigm. In the former case, the flexible link is modelled as a continuous system employing a constrained Euler–Lagrange formulation [12] which, similarly to [7], considers a limited number of mode shapes. A simulation study demonstrates the effectiveness of the disturbance–compensation control, while the comparison between the two systems highlights the potential benefits of a concurrent design and control procedure for flexible mechanisms.

The rest of the paper is organised as follows: Section 2 outlines the problem formulation and defines the class of system considered in this work, which includes the flexible pendulum-on-cart. Section 3 introduces the new energy-shaping control with adaptive disturbance–compensation and presents the stability analysis. Section 4 contains the simulation results for the flexible pendulum-on-cart, while Section 5 presents a comparison with the rigid-link pendulum employing the PRBM method. Section 6 contains concluding remarks and suggestions for future work.

2. Problem formulation

In this work we consider a class of underactuated mechanical systems with the following disturbed dynamics:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + \nabla_q V(q) = G(q)\tau - \delta \tag{1}$$

where $q \in \mathbb{R}^n$ is the generalised position, $M \in \mathbb{R}^{n\times n}$ is the positive definite inertia matrix, $C \in \mathbb{R}^{n\times n}$ is the matrix of the Coriolis and centrifugal forces, V is the potential energy. Also, G is the input matrix which, without loss of generality, is assumed of the form $G = [I^m; 0^{(n-m)\times m}]$ so that $\mathrm{rank}(G) = m < n$, and $\tau \in \mathbb{R}^m$ is the control input, while the term $\delta \in \mathbb{R}^n$ represents the disturbance. Similarly to [22,25], the proposed control law employs the full state (i.e. $q, \dot{q}$ are measurable). The problem of observer design for unmeasurable states in underactuated mechanical systems was studied in [31] and is beyond the scope of this work. The total energy of the open-loop system is $H = \frac{1}{2}\dot{q}^T M\dot{q} + V$ and the desired closed-loop equilibrium in the absence of disturbances is q_d. Partitioning q in actuated position q_a and unactuated position q_u the inertia matrix is expressed as $M = \begin{bmatrix} m_{aa} & m_{au} \\ m_{au}^T & m_{uu} \end{bmatrix}$, with $\Delta = \det(M) = m_{aa}m_{uu} - m_{au}m_{au}^T > 0$ and the disturbance as $\delta^T = [\delta_a \quad \delta_u]$.

The following assumptions are introduced for system (1) and briefly discussed:

Assumption 1: the inertia matrix M only depends on the unactuated position q_u, the block $m_{aa} > 0$ is constant, $m_{au} \neq 0$ $\forall q_u$, $\dot{m}_{uu} = c_u + c_u^T$, and $m_{uu} > 0$ since $\Delta > 0$

Assumption 2: the matrix C has the following structure $C = \begin{bmatrix} c_a & c_{au} \\ 0 & c_u \end{bmatrix}$, with $c_a,\ c_u > 0$.

Assumption 3: the potential energy can be partitioned as $V = V_a(q_a) + V_u(q_u)$

Assumption 4: there exist a function V_N so that $\dot{V}_N = -m_{au}\dot{q}_u$

Assumption 5: the system has dimension $n = 2m$