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A mixed-form solution to the macroscopic elastic properties of 2D triaxially braided composites based on a concentric cylinder model and the rule of mixture

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ABSTRACT

Understanding the elastic properties of 2D triaxially braided composites (2DTBCs) is fundamental for further analysis and design of structural components made from 2DTBCs. Based on a concentric cylinder model (CCM) and the rule of mixture (RoM), we reformulated the global stiffness of 2DTBCs for the anisotropic carbon fiber reinforcement to replace the currently available formulations for the isotropic glass fiber reinforcement with a replacement scheme. First, the global stiffness calculated from CCM, Quek's model and Shokrieh's model was compared for clarification. It was found that Shokrieh's model had the relatively worse performance under the same conditions. Second, the performances of Shokrieh's model, CCM, Quek's model and CCM + RoM (Reuss-type and Voigt-type) were compared using available experiments under various braiding configurations and our experiments on the hybrid carbon/glass fiber reinforced 2DTBCs, which revealed that Quek's model and CCM + RoM (Reuss-type) could yield equivalently great results, and CCM + RoM (Reuss-type) performed even better in some cases.

1. Introduction

As representatives of lightweight materials with high stiffness and strength, fiber-reinforced/braided materials have been widely investigated for their applications in the fields of aerospace, automotive, military, energy, sports, *etc*. Generally, these materials include 2D [1–4] and 3D [5–8] braided materials. The mechanical properties of braided materials are of interest all the time and the modelling and application of braided materials have been reviewed [9–11]. In some applications, the elastic properties of 2D triaxially braided composites (2DTBCs) have drawn much attention since it is fundamental for further investigations of the performance of structural components made from 2DTBCs, such as structural static [12,13] and dynamic [14,15] responses, failure analysis [16,17], damage propagation [18,19], *etc*. For example, Masters et al. [20] firstly investigated the elastic properties of 2DTBCs experimentally, and built an analytical model based on the rule of mixture (RoM). The effects of various braiding parameters, such as braid angle, yarn size and axial yarn content, were tested for the elastic properties of 2DTBCs [21]. However, the effect of the crimp angle of the undulating fibers had been ignored. Using experiments, Phoenix [22] demonstrated that the crimp angle could also influence the elastic properties of the braided material as well as the braid angle. Usually, a 2DTBC consists of four constituents: one axial tow (*i.e.*, yarn), two braid tows and one matrix. The macroscopic stiffness of the 2DTBC is obtained by the volume average of each constituent. For instance, based on the unit cell, Byun [23] developed an analytical model to predict a complete set of the engineering constants of a 2DTBC which was made of carbon fiber reinforced polymer (CFRP). However, in this model, a nontrivial geometric model must be established first to obtain the fiber volume fractions, and the fiber packing fractions must be specifically given or measured in advance. Yan and Hoa [24] used a similar geometric model to obtain the fiber volume fractions and calculated the global stiffness of 2DTBCs based on an energy model. Moreover, Quek et al. [25] suggested to obtain the volume fractions of each constituent

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by computer aided design (CAD) tools, and they formulated an analytical solution for the complete stiffness matrix of 2DTBCs with the isotropic glass fiber reinforcement. On the contrary, Shokrieh and Mazloomi [26] decomposed the 2DTBC into three layers with identical thickness, including one layer of axial fibers with some matrix, and two layers of braid fibers with some other matrix. The global stiffness of the 2DTBC was calculated as the direct summation of that in each layer.

It should be noted that Quek et al. [25], and Shokrieh and Mazloomi [26] only calculated the global stiffness for 2DTBCs of glass fiber reinforced polymers (GFRPs), where the glass fibers were simply dealt with as an isotropic phase. Moreover, Quek et al. [25] provided four elastic constants (longitudinal Young's modulus E_{11}, longitudinal Poisson's ratio ν_{12}, longitudinal shear modulus G_{12}, and transverse shear modulus G_{23}) of the local stiffness of the transversely isotropic axial tow through a concentric cylinder model (CCM) [27–29], with the fifth constant (transverse Young's modulus E_{22}) obtained through a modified version of RoM [25]. On the other hand, the result from Shokrieh's model [26] seemingly agreed better with the experiments [25] than that from Quek's model [25]. However, the assumptions on the volume fractions and geometrical properties of the undulating tows in Shokrieh's model were different from those of the experiments. Thus, Shokrieh's model needs to be further validated in details. To clarify the above issues, the global stiffness of GFRP-2DTBCs from Shokrieh's model will be recalculated with the correct volume fractions of each tow for the comparison with the experiments and Quek's model.

Furthermore, since Quek's model was essentially a mixed form of CCM and a modified version of RoM, it would be interesting to compare Quek's model with CCM. In this case, we calculated and compared the global stiffness of the CFRP-2DTBC from CCM, Quek's model and CCM + RoM (Reuss-type and Voigt-type) with the available experiments [30] under different braiding configurations and our experiments on the hybrid carbon/glass fiber reinforced 2DTBCs as well. It turned out that Quek's model and CCM + RoM (Reuss-type) performed the best, and CCM + RoM (Reuss-type) was surprisingly better in some cases. This unexpected result will be discussed in details in this work. In addition, based on a replacement scheme [31,32], all models mentioned in this work were reformulated for the anisotropic carbon fiber reinforcement instead of the isotropic case used for the GFRP-2DTBC, *e.g.*, that in the original CCM and Quek's model, so that the global stiffness of CFRP-2DTBC can be directly calculated for other purposes, e.g., static analysis, dynamic or impact analysis and strength analysis [16,33,34].

2. Methodology

The analysis of the 2DTBC is usually based on a representative unit cell (RUC), as shown in Fig. 1. The axis-1 denotes the direction of the axial tow, the axis-2 is the transverse direction and the axis-3 is vertical to the plane of the 2DTBC. First, the 2DTBC is considered to consist of four constituents, one axial tow, two braid tows and one matrix (Fig. 2). Each tow is composed of a group of fibers and some amount of binding

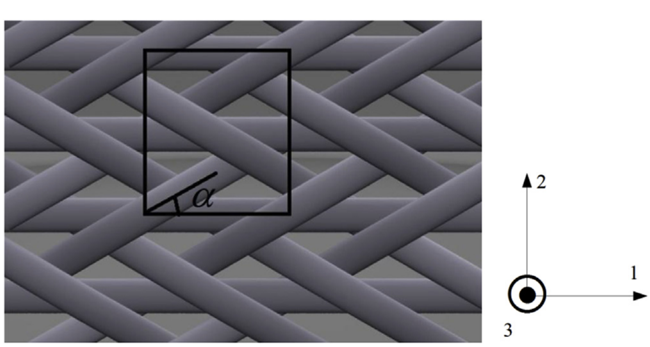

Fig. 1. A representative unit cell as shown in the black-line box of 2DTBC with braid angle α

matrix, which is closely attached to the fibers. Since only the elastic properties of the 2DTBC are considered here, the fibers and matrix are assumed to be perfectly bonded and all interfaces are considered to be coherent. The stiffness of each tow should be determined first. For most cases [23–26], the stiffness of the axial tow is calculated in advance because this tow can be simply assumed to be straight. For the GFRP-2DTBC, both the original CCM and Quek's model have formulated the five independent elastic constants of the local stiffness of the transversely isotropic axial tow. For the CFRP-2DTBC, the axial tow is also considered to be transversely isotropic and the five elastic constants can be obtained by a replacement scheme [31,32]. Thus, we will progressively formulate the macroscopic elastic properties of CFRP-2DTBC in the following sections. Also, we will review Quek's model and Shokrieh's model, to notify their different points in Sections 2.2 and 2.4, respectively.

2.1. Local stiffness of CCM

In the original CCM, the five elastic constants are transverse bulk modulus k_{23}, longitudinal Young's modulus E_{11}, longitudinal Poisson's ratio ν_{12}, longitudinal shear modulus G_{12}, and transverse shear modulus G_{23}. The first four are based on the two-phase CCM [28,29]. Here, we rewrote them for the CFRP-2DTBC instead of the isotropic case of the GFRP-2DTBC in the original CCM and Quek's model by a replacement scheme [31,32] (Isotropic matrix is always assumed):

$$k_{23} = k_m + \frac{V_f}{1/(k_{23f} - k_m) + V_m/(k_m + G_m)}, \tag{1}$$

$$E_{11} = E_{11f} V_f + E_m V_m + \frac{4 V_f V_m (\upsilon_{12f} - \upsilon_m)^2}{V_f/k_m + V_m/k_{23f} + 1/G_m}, \tag{2}$$

$$\upsilon_{12} = \upsilon_{12f} V_f + \upsilon_m V_m + \frac{V_f V_m (\upsilon_{12f} - \upsilon_m)(1/k_m - 1/k_{23f})}{V_f/k_m + V_m/k_{23f} + 1/G_m}, \tag{3}$$

$$G_{12} = G_m \frac{G_{12f}(1 + V_f) + G_m V_m}{G_m(1 + V_f) + G_{12f} V_m}, \tag{4}$$

where V is the volume fraction, $V_f + V_m = 1$, where the local fiber volume fraction V_f is also called the fiber packing fraction [23,24], k is the transverse bulk modulus, E is the Young's modulus, ν is the Poisson's ratio, and G is the shear modulus. Subscripts m and f represent the matrix and fiber, respectively.

In Fig. 3, the original two-phase CCM assumes that any transverse sections of the axial and braid tows are filled with composite cylinders of different radii, in which the fiber volume fractions and cylinder properties are the same. It requires that the absolute size of the fibers must vary down to infinitesimal to achieve the volume filling configuration. Unfortunately, transverse shear modulus or transverse Young's modulus cannot be derived due to the boundary conditions [28]. However, by combining with the generalized self-consistent method, Christensen [27] extended the original two-phase CCM into a three-phase CCM, and the transverse shear modulus were solved through a quadratic equation:

$$A\left(\frac{G_{23}}{G_m}\right)^2 + B\left(\frac{G_{23}}{G_m}\right) + D = 0. \tag{5}$$

The coefficients in the quadratic equation were given as:

$$\begin{aligned} D = {} & 3V_f V_m^2\left(\frac{G_{23f}}{G_m} - 1\right)\left(\frac{G_{23f}}{G_m} + \zeta_f\right) \\ & + \left[\frac{G_{23f}}{G_m}\zeta_m + \left(\frac{G_{23f}}{G_m} - 1\right)V_f + 1\right] \\ & \times \left[\frac{G_{23f}}{G_m} + \zeta_f + \left(\frac{G_{23f}}{G_m}\zeta_m - \zeta_f\right)V_f^3\right], \end{aligned} \tag{6}$$