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Full-description of integer Lau effect with arbitrary amplitude diffracting grating

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ABSTRACT

A complete theoretical explanation of the integer Lau effect is presented for arbitrary transmission function of diffracting grating based on Fresnel diffraction theory and Fourier series representation of the grating function. It is shown that, in the case of amplitude diffracting grating, for some gratings separation distances which are equal to an even multiple of an elementary distance the intensity distribution function at output plane in Lau setup are similar to the squared modulus of the diffracting grating. For the gratings separation distances equal to an odd multiple of the elementary distance the output intensity functions are also similar to the squared modulus of the diffracting grating but with only its even frequencies. Binary gratings are treated as an important case. Performed simulations and experiments verify correctness of this theoretical analysis.

1. Introduction

In 1948 Lau observed well-defined interference patterns for proper separations between two gratings in a setup shown in Fig. 1 [1]. Spatially incoherent light illuminates the two amplitude gratings with mutually parallel bars. A well-defined fringe pattern appears at focal plane of the lens L when the proper relationship between the gratings separation distance z, the first grating period, the second grating period p, and mean light wavelength λ is satisfied.

Lau effect have been used for different applications, such as interferometry [2,3], collimation testing [4], refractometry [5], measurement of focal length [6] using phase shifting Lau phase interferometry [7], tilt angle [8], and surface profilometry [9,10].

Along with these applications, several researchers have explained this effect by various approaches. These approaches are using Fresnel diffraction directly [11–13], grating imaging [14,15], coherence theory [16–18], optical transfer function [19], incoherent superposition of self-images produces by the second grating [20]. A good reference on this subject is Patorsky review article [21].

This manuscript is restricted only to integer Lau effect, i.e. cases where the distance z, in Fig. 1, is an integer multiple of the basic Lau distance $z_L = p^2/(2\lambda)$, while a fractional Lau effect also exists when z is a rational multiple of z_L. But, on the other hand, this paper gives a more neat treatment for the integer Lau effect in comparison to the previous papers.

Several setups have been considered in papers describing the Lau effect. In this work we consider the setup which is reported by Lau, as shown in Fig. 1. In the next section intensity distribution at observation plane is calculated directly by using Fresnel diffraction integral which is based on the use of the Fresnel transform [22]. The two gratings are considered amplitude type. The first grating is assumed with narrow opening and the second grating with arbitrary amplitude transmission function. Analysis of the intensity pattern is covered by Section 3. Lau effect with binary gratings, as an important type of gratings, are treated in Section 4. In the last section some performed simulations and experiments are reported.

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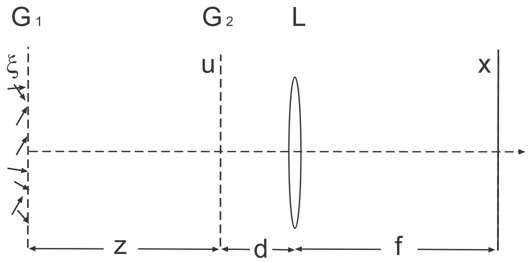

Fig. 1. System of Lau effect, G_1 and G_2 are gratings, L is a lens with focal length *f*.

2. Intensity pattern at image plane

Fig. 1 illustrates the setup of Lau's experiment. Two amplitude diffraction gratings, G_1 and G_2, in tandem are illuminated by an extended light source. Intensity pattern forms at the back focal plane of the lens L. We want to specify characterizations of the pattern as clear as possible. In this treatment we imagine three assumptions: (a) paraxial approximation holds, (b) the grating G2 has infinite extent, and (c) the positive lens does not act as a low band filter. These assumptions are justifiable as long as we concern with gratings of conventional periods and lenses of conventional diameters.

Let $t_A(u)$ is amplitude transmission function of G_2. With a point source on ξ plane at ξ_0, the field just behind the grating G_2, ignoring a constant coefficient, is

$$U(u,\xi_0)=\exp\left(\frac{\mathrm{j\,k}}{2z}u^2\right)\exp\left(-\frac{\mathrm{j\,k}}{z}u\xi_0\right)t_A(u), \tag{1}$$

where $k = 2\pi/\lambda$. The field at focal plane of the lens becomes [23]

$$U(x,\xi_0)=\frac{\exp\left[\frac{\mathrm{jk}}{2f}\left(1-\frac{d}{f}\right)x^2\right]}{j\lambda f}\int U(u,\xi_0)\exp\left(-\frac{\mathrm{jk}}{f}\mathrm{ux}\right)\mathrm{du}, \tag{2}$$

where d is separation between the grating G_2 and the lens, and f is focal length of the lens. Eq. (1) put into Eq. (2) gives

$$\begin{aligned} U(x,\xi_0) &= \frac{\exp\left[\frac{\mathrm{jk}}{2f}\left(1-\frac{d}{f}\right)x^2\right]}{-\lambda^2 \mathrm{f\,z}}\exp\left(\frac{\mathrm{jk}}{2z}\xi_0^2\right) \\ &\quad\times\int \exp\left(\frac{\mathrm{j\,k}}{2z}u^2\right)\exp\left(-\frac{\mathrm{j\,k}\xi_0}{z}u\right)t_A(u)\exp\left(-\frac{\mathrm{j\,kx}}{f}u\right)\mathrm{du}. \end{aligned} \tag{3}$$

$t_A(u)$ can be written as

$$t_A(u)=\sum_{n=-\infty}^{\infty}a_n\exp\left(\frac{j2\pi\mathrm{nu}}{p}\right). \tag{4}$$

By using Eq. (4), ignoring an unimportant coefficient, we have

$$\begin{aligned} U(x,\xi_0) &= \sum_{n=-\infty}^{\infty}a_n\int\exp\left(\frac{\mathrm{j\,k}}{2z}u^2\right)\exp\left(-\frac{\mathrm{j\,k}\xi_0}{z}u\right)\exp\left(-\frac{\mathrm{jkx}}{f}u\right)\exp\left(\frac{j2\pi\mathrm{nu}}{p}\right)\mathrm{du} \\ &= \sum_{n=-\infty}^{\infty}a_n\int\exp\left[\frac{\mathrm{j\,k}}{2z}\left(u^2-2\xi_0u-\frac{2\mathrm{zx}}{f}u+\frac{2z\lambda\mathrm{nu}}{p}\right)\right]\mathrm{du} \\ &= \sum_{n=-\infty}^{\infty}a_n\int\exp\left\{\frac{\mathrm{j\,k}}{2z}\left[\left(u-\xi_0-\frac{\mathrm{zx}}{f}+\frac{n\lambda z}{p}\right)^2-\left(\xi_0+\frac{\mathrm{zx}}{f}-\frac{z\lambda n}{p}\right)^2\right]\right\}\mathrm{du}. \\ &= \sum_{n=-\infty}^{\infty}a_n\exp\left[-\frac{\mathrm{j\,k}}{2z}\left(\xi_0+\frac{\mathrm{zx}}{f}-\frac{z\lambda n}{p}\right)^2\right]\int\exp\left[\frac{\mathrm{j\,k}}{2z}\left(u-\xi_0-\frac{\mathrm{zx}}{f}+\frac{n\lambda z}{p}\right)^2\right]\mathrm{du}. \end{aligned} \tag{5}$$

The integral in the above equation is a constant and can be dropped. Therefore, after some modification and ignoring an unimportant constant,

$$U(x,\xi_0)=\sum_{n=-\infty}^{\infty}a_n\exp\left\{-\frac{\mathrm{j\,k}}{2z}\left[\frac{z}{f}\left(x+\frac{f\xi_0}{z}\right)-\frac{z\lambda n}{p}\right]^2\right\}. \tag{6}$$

Eq. (6), with omitting a phase coefficient, that is not effective in intensity calculation, can be rewritten as

$$U(x,\xi_0)=\sum_{n=-\infty}^{\infty}a_n\exp\left(-\frac{j2\pi z\lambda n^2}{2p^2}\right)\exp\left[\frac{j2\pi\mathrm{nz}}{\mathrm{pf}}\left(x+\frac{f\xi_0}{z}\right)\right]. \tag{7}$$

Then the resulted intensity at *xy* plane is