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Optical solitons in presence of higher order dispersions and absence of self-phase modulation

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ABSTRACT

This paper proposes a new model for soliton transmission through optical fibers where self-phase modulation is negligibly small and hence removed. The model however includes spatio-temporal dispersions, of second and third order, to compensate for low group velocity dispersion. The balance for sustaining solitons is provided by nonlinear dispersion terms that are considered in this model.

1. Introduction

Optical solitons are fundamental molecules that keep telecommunication industry afloat. It is these basic molecules or pulses that dictate the governing dynamics of Internet industry, social media and various other aspects. The mathematical technology of this industry is very revealing with a variety of models [1–10]. The basic mechanism of the existence of solitons is the outcome of a delicate balance that persists between group velocity dispersion (GVD) and nonlinearity. In most cases, it is the Kerr law of nonlinearity that is considered although several other forms of nonlinear media have been studied. The question now arises is that if either GVD or nonlinearity or both are negligibly small, do these solitons cease to exist? Under this situation, when nonlinearity is negligible, several different forms of governing model have been proposed and studied in the context of fiber-optic transmission. These are commonly referred to as derivative nonlinear Schrödinger's equation (DNLSE) whose three forms are known, thus far. They are occasionally labeled as DNLSE-I, DNLSE-II and DNLSE-III that are otherwise known as Kaup-Newell equation, Gerdjikov-Ivanov equation and Chen-Lee-Liu equation respectively. However, if both GVD and nonlinearity are small, the governing equation of this paper can serve as a viable model to study soliton dynamics in optical fibers, PCF and metamaterials. Today's model encompasses DNLSE-I and DNLSE-III as special cases [2,5,7–10].

Two forms of the model are studied in this paper, where the second form is a generalized version of the first form with full nonlinearity. These models will be studied by the aid of traveling wave hypothesis and exponential function approach. Bright, singular as well as combo solitons are retrieved from these two integration architectures. These two approaches are discussed in the next couple of sections after a quick introduction the model of study.

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2. Governing model

This section will display the proposed model in two forms. It is only a generalized version of the first model that will be analyzed as the second version.

2.1. Form-I

The proposed model with higher order dispersions and absence of self-phase modulation (SPM) is given as:

$$iq_t + a_1 q_{xx} + a_2 q_{xt} + i(b_1 q_{xxx} + b_2 q_{xxt}) = i[\lambda(|q|^2 q)_x + \mu(|q|^2)_x q + \theta\,|q|^2 q_x] \tag{1}$$

Here, the wave form is designated with the complex-valued function $q(x, t)$ that is dependent on two independent variables x and t which are spatial and temporal variables respectively. The first term in (1) is the temporal evolution of the wave, while a_1 is the coefficient of GVD and a_2 is the spatio-temporal dispersion (STD). Next, higher order dispersive effects are provided by third order dispersion (3OD) which is represented by b_1 and spatio-temporal 3OD (ST-3OD) comes from the coefficient of b_2. Thus, STD, 3OD and ST-3OD together compensate for low count of GVD. Also, noticeably, SPM effect is absent. Nevertheless, nonlinear effects are provided by the terms on the right hand side of the model. Here, λ gives the effect of self-steepening while μ and θ give the effect of nonlinear dispersion. Thus, even with the absence of SPM, this proposed model ideally provides the technical requirements of the existence of solitons. Bright and other soliton solutions will be recovered from this proposed model and will thus support our claim.

2.1.1. Traveling wave hypothesis

To kick off with traveling wave hypothesis, one assumes:

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where g represents the amplitude component of the wave form and

$$s = x - vt, \tag{3}$$

where v is the speed of the wave. The phase portion is split as:

$$\phi(x, t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

where κ is the frequency, ω is the wave number and θ_0 is the phase constant.

On substituting (2) into (1) leads to two components due to real and imaginary parts. The real part equation reads off to be:

$$(a_1 - a_2 v + 3b_1\kappa - 2b_2 v\kappa - \omega b_2)g'' - (\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2)g = (\lambda + \theta)\kappa g^3 \tag{5}$$

where $g' = dg/ds$ and $g'' = d^2g/ds^2$. Next, multiplying both sides of (5) by g' and integrating once, after choosing the integration constant to be zero gives:

$$2(a_1 - a_2 v + 3b_1\kappa - 2b_2 v\kappa - \omega b_2)(g')^2 = g^2\{2(\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2) + (\lambda + \theta)\kappa g^2\}, \tag{6}$$

which, after separating variables, and integrating leads to:

$$g = A_1\,\text{sech}[B_1(x - vt)], \tag{7}$$

so that the chirp-free bright 1-soliton solution to the model is:

$$q(x, t) = A_1\,\text{sech}[B_1(x - vt)]e^{i(-\kappa x + \omega t + \theta_0)}. \tag{8}$$

The amplitude and the inverse width of the soliton are respectively given by

$$A_1 = \sqrt{-\frac{2(\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2)}{(\lambda + \theta)\kappa}}, \tag{9}$$

and

$$B_1 = \sqrt{\frac{\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2}{a_1 - a_2 v + 3b_1\kappa - 2b_2 v\kappa - \omega b_2}}. \tag{10}$$

These two relations, given by (9) and (10), immediately prompts the constraint conditions

$$(a_1 - a_2 v + 3b_1\kappa - 2b_2 v\kappa - \omega b_2)(\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2) > 0, \tag{11}$$

and

$$\kappa(\lambda + \theta)(\omega + a_1\kappa^2 + b_1\kappa^3 - a_2\omega\kappa - b_2\omega\kappa^2) < 0 \tag{12}$$

for the existence of these solitons.

Next, the imaginary part equation comes out to be:

$$(b_1 - b_2 v)g''' + (b_2 v\kappa^2 + 2b_2\omega\kappa - 3b_1\kappa^2 - v - 2a_1\kappa + 2a_2 v\kappa + a_2\omega)g' = (3\lambda + 2\mu + \theta)g^2 g'. \tag{13}$$