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Correlation between geometric phase and concurrence of variable retarders of birefringent medium

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ABSTRACT

The orbital angular momentum of photon is one of the key source of quantum information. Polarized light passing through twisted special wave plates show entangled states of both spin angular momentum (SAM) and orbital angular momentum (OAM). The variable circular and linear retarders (VCR and VLR) are studied here from the view point of quantum processing of entangled states. In association with twist dependent gain of OAM, the geometric phase has been identified here both for VCR and VLR. For the application of VLR in an antenna array, we established a correlation between geometric phase and concurrence to explain the gradual flattened shape of the emergent wave by controlling the external twist of wave plates.

1. Introduction

The property of birefringence develops quantum phases in the optical material. These appeared phases may be either dynamical or geometrical or a mixture of both. There are four types of geometric phases (GP) [1]. In the context of cyclic change of polarization, we are interested here Pancharatnam [2] phase $\Omega/2$,where Ω is the solid angle enclosed by the path of the light ray on the Poincare sphere. van Enk [3] pointed out that Pancharatnam phase in mode space is associated with spin angular momentum (SAM) transfer of light and optical medium. The GP for OAM of polarized photon has been studied theoretically by Padgget [4] in Poincare sphere and experimentally by Galvez et al. [5] in mode space.

The physical mechanism of different kind of GP's originate from spin or orbital angular momentum of polarized photon. Light beam having well defined SAM and OAM are described by the polarization and wave front around the beam axis respectively [6]. For a paraxial beam having an azimuthal phase dependence $exp(il\varphi)$, the eigenstates in OAM Hilbert space are denoted by $|l\rangle$, correspond to the integer l, where φ is the azimuthal angle around the beam axis [7–9]. It is now known OAM could be further of two types: internal and external where the later is associated with external twist of the medium.

In contrast to the previous idea, recently it has found that OAM and SAM could couple through specific complex media where change of SAM modifies the OAM and vice versa. Marrucci et al. proposed that in anisotropic inhomogeneous media, the variation of SAM occurring from the mediums birefringence gives rise to the appearance of OAM, arising from medium's inhomoginity [10]. The OAM beams are generated by a kind of birefringent plate known as “q-plates” [11] which have very fruitful applications in the classical and quantum regime. Agarwal also pointed out [12] that that q plate is the device which produces entangled state of orbital degree of freedom (OAM) and spin degrees of freedom (SAM) resulting the mutual involvement of degrees of freedom of optical

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medium. The strength of entanglement can be measured through physical quantity called concurrence. This measured term has close relationship with geometric phase [13] for entangled state of spin $-1/2$ particles.

Although some work has been done on VCR and VLR in connection with GP but its effects on OAM generation is not directly studied [14,15] as yet. Work reported in this literature on variable retarders are composed of material possessing circular and linear birefringence. We here like to study the entangled states from twisted variable circular retarder (VCR) and variable linear retarder (VLR) to find their corresponding geometric phases. As an application of VLR in antenna array, in the context of quantum information processing, we will study the variation of GP with the concurrence by controlling the external twist of wave plates.

2. Matrix representation of birefringent media

The optical properties such as birefringence and dichroism of a homogeneous medium vary with distance as pointed out by Jones [16].The property of birefringence of this optical medium can be represented by the differential matrix N. At a particular position z of optical media, Jones showed that the spatial variation of the polarization matrix $M = \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}$ develops the N matrix. For a fixed wavelength through an infinitesimal distance within the optical element the propagation of the polarized light ε can be represented by [17]

$$\frac{d\varepsilon}{\mathrm{dz}} = \frac{\mathrm{dM}}{\mathrm{dz}}\varepsilon_0 = \frac{\mathrm{dM}}{\mathrm{dz}}M^{-1}\varepsilon = N\varepsilon \tag{1}$$

where it is evident that N is the operator that determines dM/dz from polarization matrix M.

$$N = \frac{\mathrm{dM}}{\mathrm{dz}}M^{-1} = \begin{pmatrix} n_1 & n_2 \\ n_3 & n_4 \end{pmatrix} \tag{2}$$

Eight optical properties through eight constants are required to specify [17] the real and imaginary parts of four elements of N matrix identifying absorption, birefringence, dichroism along linear and circular directions. The evolution of light ray vector through medium N is equivalent to the cyclic change of the state vector during the passage of infinitesimal distance dz of the optical medium. A natural twist for an elementary angle $d\alpha = \Omega dz$ will be realized by the instantaneous ray vector $\vec{p}$ through dz by $\frac{d\vec{p}}{\mathrm{dz}} = \vec{\Omega} X \vec{p}$.

A polarized light traveling in the z direction can be written as a two component spinor in an arbitrary superposition $\alpha|0\rangle + \beta|1\rangle$, where $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are the standard unit of quantum information. In the spinorial representation, the incident polarized photon can be written as [18,19] in terms of elementary qubits and spherical coordinates.

$$|\psi\rangle = e^{i\varphi}\cos\theta/2|0\rangle + \sin\theta/2|1\rangle \approx e^{i\varphi}\begin{pmatrix} Y_1^0 \\ Y_1^1 \end{pmatrix} \tag{3}$$

On spherical geometry parameterized by (θ, ϕ) this spinor has been represented by spherical harmonics Y_l^m for l=1 and $m = 1, 0, -1$ which represent the conventional Poincare sphere and equivalently as OAM sphere for $l = 1$. The polarization matrix $M = (|\psi\rangle\langle\psi| - 1/2)$ [20] can be constructed as

$$M = \begin{pmatrix} Y_1^0 Y_1^{*0} - 1/2 & Y_1^0 Y_1^{*1} \\ Y_1^1 Y_1^{*0} & Y_1^1 Y_1^{*1} - 1/2 \end{pmatrix} \tag{4}$$

whose every element is spherical harmonics for $l = 1$.

$$M = \frac{1}{2}\begin{pmatrix} Y_1^0 & Y_1^1 \\ Y_1^{-1} & -Y_1^0 \end{pmatrix} \tag{5}$$

For the conjugate state

$$|\widetilde{\psi}\rangle = e^{-i\varphi}\begin{pmatrix} Y_1^0 \\ -Y_1^{-1} \end{pmatrix} \tag{6}$$

the corresponding polarization matrix

$$\widetilde{M} = \frac{1}{2}\begin{pmatrix} -Y_1^0 & Y_1^{-1} \\ Y_1^1 & Y_1^0 \end{pmatrix} \tag{7}$$

The polarization matrix for this case is parameterized by (θ, ϕ) lies on the conventional Poincare sphere and equivalent as OAM sphere for $l = 1$. SAM of polarized photon is associated with optical polarization. Helicity is considered as a measure of OAM parameterized by additional variable χ for which SAM sphere would be in (θ, χ). Another parameter χ for helicity is included that extends the Poincare sphere to (θ, ϕ, χ) whose picture is seen in Fig. 1. Hence the parameter for helicity, χ changes with the change of polarization of light. With every value of OAM, there are two different polarization state for which there exists 2 to 1 correspondence between SAM and OAM sphere. In every OAM sphere there exists two SAM hemi-sphere. Further study is needed to evaluate the