Optik - International Journal for Light and Electron Optics 174 (2018) 508–512

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Short note

Cnoidal and solitary waves of a nonlinear Schrödinger equation in an optical fiber

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ARTICLE INFO

PACS:
05.45.Yv
04.20.Jb
42.65.Tg

Keywords:
Nonlinear Schrödiger equation
Optical fiber
Cnoidal soliton
Telecommunication systems

ABSTRACT

This paper presents new exact analytical solutions of a nonlinear Schrödinger equation with a cubic–quintic nonlinearity and in presence of a couple of perturbation terms. This equation describes the dynamics of soliton propagation through an optical fiber. Several solutions are found without applying the computer codes and by considering the integration constant. The solutions are bright, dark and cnoidal solitons. These solutions may have significant applications in telecommunication systems where solitons are used to codify or for the transmission of data. The method used here is very effective and powerful and can be applied to other types of nonlinear equations.

1. Introduction

Nonlinear optic is today one of the areas which attracts the attention of many researchers. This attention is motivated by the fact that optic medium can be used to transport energy or information in telecommunication for example. The dynamics of optic mediums is generally described by the nonlinear Schrödinger equation, the Ginzburg–Landau equation and the Korteweg de Vries equation. The solutions of these equations are most often solitons. The concept of soliton is a fascinating notion which attracts the attention of the great majority of researchers. The beginning of soliton physics is dated back to the month of August 1834 when John Scott Russell observed the great wave of translation [1]. After that, solitons research has been conducted in diverse fields such as meteorology, nonlinear electrical lines, biology, cosmology and optical fibers, to cite a few. Optical solitons have promising potential to become principal information carriers in telecommunication due to their capability of propagating long distance without attenuation and changing their shapes. The pioneering works of Hasegawa and Tappert [2], who predicted solitons theoretically, and Mollenauer, Stolen, and Gordon [3], who observed them experimentally, made solitons a realistic tool for this cause.

This paper studies the dynamics of soliton propagation through an optical fiber with a cubic–quintic nonlinearity and in presence of a couple of perturbation terms. This optical fiber is governed by a nonlinear Schrödinger's equation. In the literature, there are many powerful methods which have been proposed to obtain exact analytic solutions of nonlinear partial differential equations [4–31]. Our objective here is to find exact solutions of the model under consideration without applying the computer codes. In the continuation, we will present the model. After that, we will deal with the investigation of solutions and we will end by a conclusion.

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https://doi.org/10.1016/j.ijleo.2018.08.094
Received 22 March 2018; Received in revised form 29 June 2018; Accepted 23 August 2018

2. Presentation of the model

In this section, we briefly outline the model studied. This model is governed by a perturbed nonlinear Schrödinger's (NLS) equation given by Eq. (1):

$$i\psi_t + a_1\psi_{xx} + (a_3|\psi|^2 + a_5|\psi|^4)\psi - i\alpha\psi_x - i\beta(|\psi|^2\psi|)_x = 0, \tag{1}$$

where a_1, a_3, a_5, α and β are real constants related to the group velocity dispersion, the cubic nonlinearity, the quintic nonlinearity, the inter-modal dispersion and the self-steepening effect respectively. Now, we consider that the field function $\psi(x, t)$ of Eq. (1) can be taken as follows

$$\psi(x, t) = A(\xi)e^{i\theta}, \quad \text{where} \quad \theta = f(\xi) - \omega t \quad \text{and} \quad \xi = \mathrm{kx} - \lambda t. \tag{2}$$

Substituting Eq. (2) into Eq. (1), and separating real and imaginary parts, we obtain

$$[(\lambda + \alpha k)f' + \omega - a_1k^2f'^2]A + 3k\beta A^3f' + a_1k^2A'' + a_3A^3 + a_5A^5 = 0 \tag{3}$$

and

$$k^2a_1f''A + (2k^2a_1f' - \alpha k - \lambda)A' - 3k\beta A^2A' = 0. \tag{4}$$

Under the constraint

$$f' = \frac{\lambda + \alpha k}{2k^2a_1} + \frac{3\beta}{4\mathrm{ka}_1}A^2, \tag{5}$$

Eq. (4) is satisfied identically. Introducing Eq. (5) into Eq. (3), we obtain

$$a_1k^2A'' + \left(\omega + \frac{(\lambda + \alpha k)^2}{4k^2a_1}\right)A + \left(\frac{3\beta(\lambda + \alpha k)}{2\mathrm{ka}_1} + a_3\right)A^3 + \left(\frac{27\beta^2}{16a_1} + a_5\right)A^5 = 0. \tag{6}$$

Eq. (6) is an elliptic differential equation describing the evolution of the wave amplitude in the optical fiber. In what follows, we present novel solutions. We also give the conditions for which these optical soliton solutions exist.

3. Exact solutions: cnoidal and solitary waves

In order to obtain the exact solutions of the model studied, we multiply Eq. (6) by A' and integrating with respect to ξ; we get

$$(A')^2 = -\frac{1}{a_1k^2}\left(\omega + \frac{(\lambda + \alpha k)^2}{4k^2a_1}\right)A^2 - \frac{1}{2a_1k^2}\left(\frac{3\beta(\lambda + \alpha k)}{2\mathrm{ka}_1} + a_3\right)A^4 - \frac{1}{3a_1k^2}\left(\frac{27\beta^2}{16a_1} + a_5\right)A^6 + C, \tag{7}$$

where C is an arbitrary constant of integration. Solution of Eq. (7) can be constructed by means of some of the methods presented above. However, we can obtain the general solution of this equation by using only an analytical resolution.

Supposing

$$A(\xi) = \pm\frac{1}{\sqrt{y(\xi)}}, \tag{8}$$

Eq. (7) becomes

$$(y')^2 = -2(\mathrm{Cy}^3 - \mathrm{ay}^2 - \mathrm{by} - \sigma), \tag{9}$$

with $a = -\frac{2}{a_1k^2}\left(\omega + \frac{(\lambda+\alpha k)^2}{4k^2a_1}\right)$, $b = -\frac{1}{a_1k^2}\left(\frac{3\beta(\lambda+\alpha k)}{2\mathrm{ka}_1} + a_3\right)$ and $\sigma = -\frac{2}{3a_1k^2}\left(\frac{27\beta^2}{16a_1} + a_5\right)$.

3.1. Case 1: $C = 0$

When $C = 0$, $a > 0$ and $4a\sigma - b^2 > 0$, we have the following integral

$$\int \frac{\mathrm{dy}}{\sqrt{\mathrm{ay}^2 + \mathrm{by} + \sigma}} = \sqrt{2}(\xi - \xi_0), \tag{10}$$

from which we have

$$\sinh^{-1}\left(\frac{2\mathrm{ay} + b}{\sqrt{4a\sigma - b^2}}\right) = \sqrt{2a}(\xi - \xi_0). \tag{11}$$

Eq. (11) leads to

$$y(\xi) = \frac{\sqrt{4a\sigma - b^2}}{2a}\sinh(\sqrt{2a}(\xi - \xi_0)) - \frac{b}{2a}. \tag{12}$$

Using Eq. (8), we have