Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 178 (2018) 59–81

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Large strategic dynamic interactions ☆

Ehud Kalai, Eran Shmaya *

Kellogg School of Management, Northwestern University, United States of America

Received 26 July 2017; final version received 27 July 2018; accepted 2 August 2018
Available online 27 August 2018

Abstract

The paper presents a model of large strategic dynamic interactions in an environment with uncertain fundamentals. The interaction is among a large finite group of interdependent players, diversified in their preferences and information. We study an imagined-continuum equilibrium, a behavioral hybrid of games with a continuum of players and finite games. This equilibrium enables simple Bayesian reasoning and admits natural Markov-perfect equilibria. In addition, we establish bounds on the probabilistic discrepancies between players' beliefs that are derived from the continuum model and the actual finite reality.

JEL classification: C72; C73

Keywords: Anonymous games; Nash equilibrium; Repeated games; Large games; Bayesian equilibrium; Markov equilibrium

☆ *Acknowledgment*: We are grateful to Francesco Vitiello for correcting a mistake in an earlier version of the paper. Early versions of the paper also benefitted from presentations at Tel Aviv University, The Hebrew University, Northwestern University, University of California San Diego, Microsoft Research New England, Harvard University, SUNY at Stony Brook, Duke University, Stanford University, University of Pennsylvania, California Institute of Technology, University of California Berkeley, Oxford University, and more. Parts of the paper were presented in the 2015 Nancy L Schwartz Memorial Lecture at Northwestern University.

* Corresponding author.
E-mail addresses: kalai@kellogg.northwestern.edu (E. Kalai), e-shmaya@kellogg.northwestern.edu (E. Shmaya).

https://doi.org/10.1016/j.jet.2018.08.001

1. Introduction

1.1. Overview

Large dynamic interactions play major roles in economic, political, online and other social systems. Typically, such an interaction involves a large finite number of participants and uncertain fundamentals that are an essential part of the functioning of the system. These uncertain fundamentals may have different effects on the preferences and information of the individual participants.

Papers that deal with large interactions cover a variety of theoretical issues and applications. Examples include markets (Novshek and Sonnenschein, 1978), bargaining (Mailath and Postlewaite, 1990), auctions (Olszewski and Siegel, 2016; Rustichini et al., 1994), voting (Feddersen and Pesendorfer, 1997; Myerson, 2000), electronic commerce (Gradwohl and Reingold, 2013), industrial organization and market design (Backus and Lewis, 2016; Weintraub et al., 2008).

However, these earlier papers have often been restricted to one-time interactions as opposed to ongoing interactions, to fully known fundamentals, and to perfect information and monitoring. To relax such restrictions this paper studies an uncertain large dynamic game, described as follows.

A finite large group of players plays a repeated stage game in which (1) player types, describing preferences and information, are correlated through an unknown state of fundamentals; (2) every period of play results in a random public outcome that depends on the aggregate distribution of players' actions and on the state of fundamentals, and; (3) players' period payoffs and information depend on the known public outcome combined with the players' private information.

Examples of such large dynamic games include repeated uncertain market games, in which the fundamentals may describe production and demand parameters and period outcomes may describe prices; repeated rush-hour-commute games, in which the fundamentals may describe road capacities and outcomes may describe driving times; and network-communication games, in which the fundamentals may describe technical parameters of communication devices and outcomes may describe data about their use.

We study a behavioral equilibrium concept that is tractable, yet sufficiently refined for the analysis of events in various applications. In this *imagined-continuum* equilibrium each of the n players behaves as if she is one of a continuum of players. However, the actual play-path of the game is generated in a finite n-player model.

In general terms, this paper extends the mission of the large-games literature to provide a microfoundation for large (macro) games, in Bayesian dynamic environments with bounded but highly rational players. Still, the model needs further extensions before it can be used in economic applications, as in finance and macro-economics.[1] For example, as it is now, the model cannot address issues of economic growth and shocks.

As an example, we use the model to describe a Markov-perfect equilibrium in a repeated Cournot game, when the traders have diverse preferences and interdependent information about uncertain fundamentals.

[1] Krusell and Smith (1998) present a different use of moments of aggregate distributions to model boundedly rational players in a large interaction, and Acemoglu and Jensen (2015) present a general model of large dynamic economies. In a financial market, Kovalenkov and Vives (2014) compare the outcomes of "behavioral continuum" with that of strategic play with asymmetric information.