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Propagation properties of chirped Airy hollow Gaussian wave packets in a harmonic potential

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ABSTRACT

Based on the analytical expression obtained by solving the (3+1) D Schrödinger-like equation, the spatiotemporal propagation properties of the chirped Airy hollow Gaussian (CAiHG) wave packets in harmonic potential are described and discussed in detail. Results show that the folds on the isosurfaces of the CAiHG wave packets deepen with the temporal chirp parameter β increasing. The wave rings of the CAiHG wave packets can be eliminated as the distribution factor increases, which lead to the stretch of the intensity distribution along the T axis. Moreover, in HGB-like condition, with the increase of the potential width parameter, the Poynting vector and the angular momentum increase and distribute more evenly while their distributions gradually become hollow as the propagation distance increases. Besides, the quantity of the main peaks of the gradient force is exactly proportional to the potential width parameter. As the beam orders increase, the secondary peaks gradually increase while the main peaks are opposite. When $\beta \leq 0$, the gradient force rapidly decreases while the gradient force decreases first and then increases when $\beta > 0$.

1. Introduction

Solving the Schrödinger equation, the nonspreading Airy beams were theoretically derived by Berry and Balazs [1]. In 2007, the Airy beams with finite energy, which can be achieved by using a decay factor in the theory, were experimentally confirmed and reported [2]. Since then, the propagation properties of the Airy beams, such as self-healing [3], self-acceleration [4–6], and weak diffraction [7] have been concretely studied, which has led to the great applications of Airy beams on micromanipulation [8,9], laser filamentization [10], curved plasma channel generation [11] and so on. In addition, the Airy beams with a chirp parameter exhibit intriguing propagation properties when propagating through different media, such as a gradient-index medium [12], strongly nonlocal medium [13], chiral medium [14] and uniaxial crystal [15], which have aroused intense interest among optical researchers.

On the other hand, special optical beams named dark-hollow beams (DHBs) whose central intensity is zero, have attracted extensive attention because of their prominent application in applied optics and atomic optics [16–18]. Besides, the DHBs can be obtained from various generation techniques, such as optical holography [19,20], the hollow-fiber technique [21] and the anisotropic nonlinear optical method [22]. Recently, new mathematical models of beam called the hollow Gaussian beams (HGB) have described DHBs better [23]. The unusual propagation properties of HGB have attracted many optical researchers to join the HGB research field. For example, in the far field, the intensity distribution and M^2 factor of HGB were discussed in depth by Deng et al. in 2005 [24]. Soon, the analytical vector structure of HGB was exactly analyzed and described in detail [25,26]. In order to apply HGB in more specific fields, the propagation properties of HGB in the Rayleigh scattering regime [27], uniaxial crystals [28] and strongly nonlocal nonlinear media [29] were investigated. However, when the HGB is combined with the chirped Airy distribution, the new wave packets—chirped Airy hollow Gaussian (CAiHG) wave packets propagating in a harmonic potential may show many extraordinary properties due to the influence of the temporal chirp parameter and the specificity of the harmonic potential. According to the current resources available, this study has not been reported. We will explore this study in the following paper.

In the second section of this paper, the light field expression for the CAiHG wave packets propagating in the presence of a harmonic potential is deduced by solving the normalized dimensionless linear (3+1) D Schrödinger-like equation. In the third section, considering the temporal domain and the spatial domain, the propagation properties of the CAiHG wave packets in a harmonic potential are analyzed and described in detail. Finally, we summarize the entire paper in the fourth section.

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2. Model and analytical solution

In a harmonic potential, the beams specifically obey the equation

$$i\beta_0 \frac{\partial}{\partial z} u(x,y,z,\tau) + \frac{1}{2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) u(x,y,z,\tau) + \frac{1}{2}\beta_0\beta_2 \frac{\partial^2}{\partial \tau^2} u(x,y,z,\tau) - {\beta_0}^2 \frac{\delta n}{n_0}\left(x^2+y^2\right) u(x,y,z,\tau) = 0,$$

where $\beta_0 = k_0 n_0 = (\omega_0/c) n_0$, $n_0 = n(\omega_0)$, $\tau = t - \beta_1 z$, $\beta_1 = \left.\frac{d}{d\omega} n(\omega)\right|_{\omega\to\omega_0}$ is the inverse of the group speed, and $\beta_2 = \left.\frac{d^2}{d\omega^2} n(\omega)\right|_{\omega\to\omega_0}$ is the dispersion index in the case of anomalous dispersion. The above equation can be expressed in dimensionless form by introducing the new coordinates $X = x/w_0$, $Y = y/w_0$ and $Z = z/L_{dif}$, where $L_{dif} = k_0 {w_0}^2$ is the diffraction length. w_0 and $k_0 = 2\pi/\lambda_0$ denote the beam waist width and the wave number, respectively. Furthermore, the following change must be made: $T = \frac{\tau}{\tau_0}\sqrt{\frac{L_{disp}}{L_{difr}}}$. Here, $L_{disp} = {\tau_0}^2/\beta_2$ is the dispersion length. One, then, obtains the dimensionless equation

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial T^2}\right) - {k_0}^2 \frac{\delta n}{n_0}\left(X^2+Y^2\right) U = 0,$$

where $U(X,Y,T,Z)$ describes the amplitude distribution of the light field.

In order to derive the analytical expression of CAiHG wave packets propagating in a harmonic potential, the influences of diffraction and dispersion can be equalized for simpler analysis. In the harmonic potential, the paraxial CAiHG wave packets obey the normalized dimensionless linear (3+1) D Schrödinger-like equation [30]:

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial T^2}\right) - \frac{{\alpha_1}^2}{2}\left(X^2+Y^2\right) U = 0, \tag{1}$$

where α_1 stands for the potential width parameter.

Using the method of separation of variables, the solution to Eq. (1) can be defined as the following form:

$$U(X,Y,T,Z) = A(T,Z)\,\varphi(X,Y,Z). \tag{2}$$

Substituting Eq. (2) into Eq.(1), we can have the following equations:

$$i\frac{\partial A}{\partial Z} + \frac{1}{2}\frac{\partial^2 A}{\partial T^2} = 0, \tag{3}$$

$$i\frac{\partial \varphi}{\partial Z} + \frac{1}{2}\left(\frac{\partial^2 \varphi}{\partial X^2} + \frac{\partial^2 \varphi}{\partial Y^2}\right) - \frac{{\alpha_1}^2}{2}\left(X^2+Y^2\right)\varphi = 0. \tag{4}$$

The initial distribution at Z = 0 of the solution of Eq. (3) is described as $A(T,0) = Ai(T)e^{aT}e^{(i\beta-1)(T^2/{\alpha_2}^2)}$, where Ai(.) represents the Airy function and $a (0 < a \le 1)$ indicates the decay factor in T direction, α_2 denotes a dimensionless distribution factor, and β is the temporal chirp parameter. When we solve Eq. (3) using the Fourier transform and inverse Fourier transform method, the analytical equation can be expressed as:

$$\begin{aligned} A(T,Z) = {} & \frac{-i}{\sqrt{bd}} \mathrm{Ai}\left(\frac{1}{16b^2{\alpha_2}^4 d^4} - \frac{c}{2b{\alpha_2}^2 d^2}\right) \\ & \times \exp\left[-\frac{c^2}{4b{\alpha_2}^2 d^2} + \frac{c}{8b^2{\alpha_2}^4 d^4} - \frac{1}{96b^3{\alpha_2}^6 d^6}\right] \\ & \times \exp\left[i\left(\frac{1}{128{\alpha_2}^6 d^2} - \frac{a}{8{\alpha_2}^3 d} + \frac{a^2}{2}\right) Z \right. \\ & \left. + \left(a - \frac{1}{8{\alpha_2}^3 d}\right) T + \frac{1}{768{\alpha_2}^6 d^3}\right], \end{aligned} \tag{5}$$

where $b = 2iZ - 1/(i\beta-1)$. $c = -\frac{1}{8{\alpha_2}^2(i\beta-1)} + 2\alpha_2(i\beta-1)(T+iaZ) - \frac{iZ}{4{\alpha_2}^2}$ and $d = i\beta - 1$.

The solution to Eq. (4) in Fourier optics can be expressed as [31]:

$$\varphi(X,Y,Z) = -\frac{i}{2\pi} M(X,Y,Z) \iint_{-\infty}^{+\infty} \phi(\xi,\eta,0)\exp\left[iQ\left(\xi^2+\eta^2\right)\right] \times \exp\left[-iK(X\xi+Y\eta)\right] d\xi d\eta, \tag{6}$$

where $M(X,Y,Z) = K\exp\left[iQ(X^2+Y^2)\right]$, $Q = \alpha_1\cot(\alpha_1 Z)/2$, and $K = \alpha_1/\sin(\alpha_1 Z)$. It is not difficult to find that with KX and KY standing for the spatial frequencies, Eq. (4) can be shown as the 2D Fourier transform of $\phi(\xi,\eta,0)\exp\left[iQ(\xi^2+\eta^2)\right]$.

The normalized paraxial HGB at $Z = 0$ can be considered as the initial spatial solution to Eq. (4), which is expressed as:

$$\varphi(X,Y,0) = C_0(X^2+Y^2)^n \exp\left[-\left(X^2+Y^2\right)\right]. \tag{7}$$

Here, $n = 0,1,2,\ldots$ represents the order of the paraxial HGB and C_0 is a constant. By applying the Fourier transform to Eq. (7), the Fourier expression of the initial HGB is:

$$\phi(\xi,\eta,0) = C_0 n! \sum_{m=0}^{n}\left[(-1)^m \binom{n}{m}\left(\frac{1}{m!}\pi^{2m+1}\left(\xi^2+\eta^2\right)^m \times e^{-\frac{1}{4}\left(\xi^2+\eta^2\right)}\right)\right], \tag{8}$$

in which $\binom{n}{m}$ denotes a binomial coefficient.

Substituting Eq. (8) into Eq. (6), we can obtain the analytical solution to Eq. (4), which satisfies the following expression:

$$\varphi(X,Y,Z) = -\frac{iC_0}{2} M(X,Y,Z)\exp\left[-\frac{K^2\left(X^2+Y^2\right)}{1-4iQ}\right] \times n! \sum_{m=0}^{n}(-1)^m\binom{n}{m}\pi^{2m+1}\left(\frac{1}{4}-iQ\right)^{-(m+1)} L_m\left[\frac{K^2\left(X^2+Y^2\right)}{1-4iQ}\right]. \tag{9}$$

Here, $L_m(.)$ indicates the Laguerre polynomial with mth-order.

Combined Eq. (5) with Eq. (9), the exact solution to Eq. (2) can be described as:

$$\begin{aligned} U(X,Y,T,Z) = {} & \frac{-C_0}{2\sqrt{bd}}\mathrm{Ai}\left(\frac{1}{16b^2{\alpha_2}^4 d^4} - \frac{c}{2b{\alpha_2}^2 d^2}\right) M(X,Y,Z) \\ & \times n!\sum_{m=0}^{n}(-1)^m\binom{n}{m}\pi^{2m+1}\left(\frac{1}{4}-iQ\right)^{-(m+1)} L_m\left[\frac{K^2\left(X^2+Y^2\right)}{1-4iQ}\right] \\ & \times \exp\left[i\left(\frac{1}{128{\alpha_2}^6 d^2} - \frac{a}{8{\alpha_2}^3 d} + \frac{a^2}{2}\right) Z \right. \\ & \left. + \left(a - \frac{1}{8{\alpha_2}^3 d}\right) T + \frac{1}{768{\alpha_2}^6 d^3}\right], \\ & \times \exp\left[-\frac{K^2\left(X^2+Y^2\right)}{1-4iQ}\right] \\ & \times \exp\left[-\frac{c^2}{4b{\alpha_2}^2 d^2} + \frac{c}{8b^2{\alpha_2}^4 d^4} - \frac{1}{96b^3{\alpha_2}^6 d^6}\right]. \end{aligned} \tag{10}$$

Eq. (10) describes the CAiHG wave packets propagating in the harmonic potential. Based on Eq. (10), we will analyze their propagation properties in the next section.

3. Numerical analysis and discussion

According to the analytical solutions derived in the previous section, the propagation properties of the second- ($n = 2$), third- ($n = 3$) and fourth-order ($n = 4$) CAiHG wave packets in the harmonic potential are explored in detail and discussed concretely in the following.

3.1. The CAiHG wave packets in harmonic potential and in temporal domain

Controlling different temporal chirp parameters β, Fig. 1 shows snapshots describing the shape of the CAiHG wave packets in a harmonic potential, the sectional views of the intensity distribution and the graph of the maximum intensity distribution along the T axis. Because the maximum intensity distribution along the T axis with different orders is similar, we only analyze the second-order CAiHG wave packets as shown in Fig. 1(c1)–(c2) for convenience. From Fig. 1(a1)–(a3), (a4)–(a6) and (a7)–(a9), it is not difficult to see that with the temporal chirp parameter β increasing, the folds on the isosurfaces of the second-, third- and fourth-order CAiHG wave packets in a harmonic potential are