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Generalized phase-shifting digital holography using normalized phase-shifted holograms

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ABSTRACT

We propose a normalization method for the speckle-like holograms observed in digital holography. We show that the norm of phase-shifted holograms can be approximated by the same value, regardless of the phase shift, when the phase distribution of an object wave in a Fresnel diffraction field satisfies the randomness condition. We present an object-wave retrieval algorithm that uses normalized phase-shifted holograms to calculate the amplitude and phase of the object wave directly, without estimating phase-shift values. In addition, we develop a scheme to estimate a direct-current term by averaging several randomly phase-shifted holograms. The proposed method is verified through numerical simulations and optical experiments.

1. Introduction

Digital holography (DH) is used for the evaluation and quantification of the wavefront diffracted from an object. As the amplitude and phase information of the wavefront can be handled quantitatively, DH has been applied in several fields such as industrial applications, biological imaging, and information processing [1–5].

In DH, an object wave in a hologram plane is frequently retrieved using a phase-shifting method that is widely used for analyzing interference fringes [6]. The constant phase-shifting algorithm has been extensively used in DH [7], giving rise to what is generally known as phase-shifting DH (PSDH). The advantage of the phase-shifting method is its ability to retrieve the object wave without unwanted components. However, it requires an accurate phase shifter.

To alleviate the performance demands on the phase shifter, a generalized phase-shifting method that uses randomly phase-shifted interference fringes has been developed. This method was introduced by Lai et al. [8]. In their proposal, a phase-shift value was estimated from straight interference fringes on a reference mirror placed in a measurement area, and a phase map was obtained by employing the least-squares method. A statistical method for estimating random phase-shift values was proposed by Cai et al. [9]. This method used the statistical properties of a random phase distribution in a Fresnel diffraction field to estimate phase-shift values in the range from 0 to π [9–14]. Yoshikawa et al. extended this statistical method so that the phase-shift values in the range from 0 to 2π could be estimated [15,16]. Hao et al. developed another phase-shift estimation method using the extreme values of interference fringes [17]. In this method, several phase-shifted interference fringes were used to obtain the correct maximum and minimum intensities of the fringes. Vargas et al. proposed a generalized phase-shifting method based on the idea of normalizing interference fringes [18]. In this method, direct-current (DC) suppressed interference fringes were normalized, and then, a phase-shift value and a phase map were estimated using linear algebra calculation methods such as the Gram–Schmidt orthogonalization approach and principal component analysis [18–24].

In the generalized phase-shifting method, phase-shift values are estimated using phase-shifted interference fringes; then, an object wave is retrieved. Thus, this method provides advantages such as simple phase-shifting implementation, the ability to use a low-cost phase-shifter, and fast measurement using a continuous fringe-scanning technique [25]. However, as generalized phase-shifting algorithms are frequently designed to analyze interference fringes with periodic structures, they are not always applicable to the speckle-like holograms observed in DH. In particular, the algorithm that uses the normalization of interference fringes requires a certain approximation of the norm of phase-shifted interference fringes as a precondition for execution. Currently, it is unclear whether this approximation is valid for speckle-like holograms. Additionally, to normalize speckle-like holograms, it is necessary to eliminate a DC term, which may be difficult.

In this paper, we propose a normalization method for randomly phase-shifted speckle-like holograms. We show that the approximation condition on the norm can be satisfied for a speckle-like hologram

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when the phase randomness condition used in statistical generalized PSDH [16] is satisfied. Furthermore, we present an algorithm to retrieve an object wave without estimating a phase-shift value. Moreover, we develop a novel scheme to eliminate the DC term using randomly phase-shifted holograms. We verify the proposed normalization method through numerical simulations and optical experiments.

The remainder of this paper is organized as follows: In Section 2, we describe the normalization method for a randomly phase-shifted speckle-like hologram. First, we present the approximation condition on the norm of the phase-shifted hologram based on the statistical properties of a random phase distribution in a Fresnel diffraction field. Next, we describe the object-wave retrieval algorithm using the normalization approach. In addition, the scheme to eliminate the DC term is discussed. The analysis of the proposed method using numerical simulations and optical experiments is presented in Sections 3 and 4, respectively, followed by the conclusions in Section 5.

2. Method

2.1. Normalization of a randomly phase-shifted speckle-like hologram

We consider a typical PSDH system. An object is assumed to be placed at distance d from a hologram plane. The object and reference waves in the hologram plane are denoted by $O(x,y) = |O(x,y)|\exp\{j\theta(x,y)\}$ and $R(x,y) = |R|\exp(j\phi_i)$, respectively, where ϕ_i is the ith phase-shift and $j = \sqrt{-1}$. The amplitude of the reference wave is assumed to be constant. A hologram in the hologram plane is represented by

$$I_i(x,y) = |O|^2 + |R|^2 + 2|O||R|\cos(\theta - \phi_i) \quad (i = 0,1,2), \tag{1}$$

where the coordinate variable is omitted for simplicity. To normalize the hologram, it is necessary to remove the DC term from the hologram. In DH, the DC term consists of the intensities of the reference and object waves. Thus, we can consider several schemes for eliminating the DC term, as we will discuss in the next section.

After removing the DC term, according to the notation of the normalization approach, we can express the hologram as an N-dimensional vector given by

$$\widetilde{I}_i = 2|O_n||R|\cos(\theta_n - \phi_i), \tag{2}$$

where N is the total number of pixels and n is an index of the vector. Thus, the norm and normalized hologram are represented by

$$\left\|\widetilde{I}_i\right\| = 2|R|\sqrt{\sum_{n=1}^{N}|O_n|^2\cos^2(\theta_n - \phi_i)}, \tag{3}$$

$$\frac{\widetilde{I}_i}{\left\|\widetilde{I}_i\right\|} = \frac{|O_n|\cos(\theta_n - \phi_i)}{\sqrt{\sum_{n=1}^{N}|O_n|^2\cos^2(\theta_n - \phi_i)}}, \tag{4}$$

respectively.

In the normalization method for interference fringes with periodic structures, phase-shift values are obtained by assuming an approximation for the norm of phase-shifted interference fringes [18–24]. For example, in [20,22,24], it is assumed that the norm of phase-shifted interference fringes has almost the same value regardless of the phase shift. Thus, the approximation can be represented by the following expression:

$$\sqrt{\sum_{n=1}^{N}|O_n|^2\cos^2\theta_n} \approx \sqrt{\sum_{n=1}^{N}|O_n|^2\cos^2(\theta_n - \phi_i)}. \tag{5}$$

Hereafter, we refer to this condition as the norm approximation condition. For simple interference fringes with periodic structures, the norm approximation condition is satisfied if one or more periods of the interference fringes exist in a captured frame. However, it is unclear whether the speckle-like hologram satisfies the norm approximation condition.

To prove that this approximation can be used in DH, we consider the statistical properties of the Fresnel diffraction field of the object. If the distance from the object plane to the hologram plane is sufficiently large, the phase of the object wave in the hologram plane can be considered as a spatially random distribution owing to Fresnel diffraction [9–16]. The statistical properties of the random phase distribution are equivalent to those of a random distribution. This is referred to as the phase randomness condition in statistical generalized PSDH [16]. The phase randomness condition can be controlled by the incident angle of the reference wave. For example, when the incidence angle is large, a sufficient phase randomness condition can be obtained in the hologram plane regardless of object properties [16]. The probability density function for a sufficiently developed random phase distribution is represented by $P(\theta) = 1/2\pi$ for $-\pi \le \theta < \pi$ and 0 otherwise [26]. Thus, the average value of the cosine of θ reduces to approximately zero. Hence, we can consider the approximation as $\sum_{n=1}^{N}\cos\theta_n \approx 0$ for large N. When this approximation is applied to the norm calculation, the value inside the radical symbol in Eq. (5) is given by

$$\begin{aligned}\sum_{n=1}^{N}|O_n|^2\cos^2(\theta_n - \phi_i) &= \frac{1}{2}\sum_{n=1}^{N}|O_n|^2\left\{1 + \cos(2\theta_n - 2\phi_i)\right\} \\ &\approx \frac{1}{2}\sum_{n=1}^{N}|O_n|^2,\end{aligned} \tag{6}$$

where $\sum_{n=1}^{N}|O_n|^2\cos(2\theta_n - 2\phi_i) \ll \sum_{n=1}^{N}|O_n|^2$ is essentially fulfilled when θ_n is either randomly distributed (corresponding to random-phase objects) or periodically distributed (corresponding to constant-phase objects with an oblique reference wave) in the hologram. This equation suggests that the norm of the phase-shifted speckle-like hologram is reduced to a constant value that is independent of the phase shift. Therefore, we can consider that if the speckle-like hologram satisfies the phase randomness condition, it also satisfies the norm approximation condition.

2.2. Object-wave retrieval using a normalized randomly phase-shifted hologram

The object wave is retrieved as follows: First, the summation and difference of the normalized phase-shifted holograms are calculated:

$$\widetilde{I}_a = \widetilde{I}_0 + \widetilde{I}_1 = 4|O_n||R|\cos\left(\theta_n - \frac{\phi_0 + \phi_1}{2}\right)\cos\left(\frac{\Delta\phi_{01}}{2}\right), \tag{7}$$

$$\widetilde{I}_s = \widetilde{I}_1 - \widetilde{I}_0 = 4|O_n||R|\sin\left(\theta_n - \frac{\phi_0 + \phi_1}{2}\right)\sin\left(\frac{\Delta\phi_{01}}{2}\right), \tag{8}$$

where $\Delta\phi_{01} = \phi_1 - \phi_0$ is a phase-shift value that satisfies $0 < \Delta\phi_{01} < \pi$. The norm of the summation hologram is represented by

$$\begin{aligned}\left\|\widetilde{I}_a\right\| &= \sqrt{\sum_{n=1}^{N}16|O_n|^2|R|^2\cos^2\left(\theta_n - \frac{\phi_0 + \phi_1}{2}\right)\cos^2\left(\frac{\Delta\phi_{01}}{2}\right)} \\ &\approx 4|R|\cos\left(\frac{\Delta\phi_{01}}{2}\right)\sqrt{\frac{1}{2}\sum_{n=1}^{N}|O_n|^2}.\end{aligned} \tag{9}$$

Similarly, the norm of the difference hologram is given by

$$\left\|\widetilde{I}_s\right\| \approx 4|R|\sin\left(\frac{\Delta\phi_{01}}{2}\right)\sqrt{\frac{1}{2}\sum_{n=1}^{N}|O_n|^2}. \tag{10}$$

The normalized summation and difference holograms are

$$\frac{\widetilde{I}_a}{\left\|\widetilde{I}_a\right\|} = \frac{|O_n|}{M}\cos\left(\theta_n - \frac{\phi_0 + \phi_1}{2}\right), \tag{11}$$

$$\frac{\widetilde{I}_s}{\left\|\widetilde{I}_s\right\|} = \frac{|O_n|}{M}\sin\left(\theta_n - \frac{\phi_0 + \phi_1}{2}\right), \tag{12}$$