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Nonparaxial, quasihomogeneous electromagnetic sources and their wide-angle far fields

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ABSTRACT

We consider random, three-component, planar, quasihomogeneous electromagnetic sources and the nonparaxial far fields which they produce. We show that by measuring the far-field spectral polarization matrix on a spherical surface throughout a half-space enables to reconstruct the low spatial-frequency parts of the degrees of correlation between the parallel source field components. Further, detecting, in addition, the far-zone angular cross-spectral density matrix for all pairs of directions with significant intensity allows to deduce the low spatial-frequency contributions to the spectral densities of the source field components as well as the degrees of correlation between the orthogonal components. The validity of the formalism is demonstrated by considering a three-component, electromagnetic Gaussian Schell-model source. The results are important for the research on electromagnetic wide-angle radiators such as many thermal and fluorescent sources and light-emitting diodes.

1. Introduction

Partially coherent light sources whose spatial degree of coherence depends only on the separation of the two points are frequently encountered in optics and are known as Schell-model sources [1,2]. A special class of Schell-model sources are the quasihomogeneous sources [3–8], whose intensity varies much slower with position than the degree of coherence changes with the separation of the observation points. An important and useful property of scalar quasihomogeneous sources is that the low spatial-frequency parts of the degree of coherence and the source intensity are specified by the far-field intensity distribution and angular coherence, respectively [7,8]. Recently, also electromagnetic, beam-like quasihomogeneous sources have been started to consider and the relationship between the far-field and source coherence has been studied [9–12].

While the quasihomogeneous electromagnetic sources have attracted considerable interest in optics research, their nonparaxial, three-component versions have not been analyzed much. As far as we know, the only exception is the case of quasihomogeneous isotropic sources [13]. In this work, we study a general, three-component, electromagnetic quasihomogeneous source in the space-frequency domain and analyze the wide-angle far field it generates. In particular, we show that the detection of the far-field polarization matrix on a spherical surface throughout a half-space provides the degrees of correlation between the parallel source field components. Additional information included in the far-field angular cross-spectral density matrix leads to the spectral densities of all three source field components and the degrees of correlation between the orthogonal components. This is in contrast to the scalar quasihomogeneous sources for which the source correlations are given by the far-field intensity distribution alone. In addition, unlike the reciprocity relations derived for electromagnetic beams which concern the traces of the polarization and cross-spectral density matrices [9–11], the formalism we present allows to deduce the nine individual correlation coefficients and the three spectral densities. Our results find use in the research of various thermal [14,15] and fluorescent emitters and light-emitting diodes [16] that can be regarded as quasihomogeneous sources.

This paper is organized as follows: in Section 2, we first recall the general connection between the electromagnetic source coherence and the far field the source produces. In Section 3, the theory is applied to three-component quasihomogeneous sources. In Section 4, we study what far-field information is required to reconstruct the intensities of the source field components and the degrees of correlation among the components. Section 5 exemplifies the formalism in terms of a nonparaxial Gaussian Schell-model source. The main results are summarized in Section 6.

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2. Electromagnetic planar source and its far field

Consider a planar, finite, random, three-component electromagnetic source in the plane $z = 0$ with the geometry and notations shown in Fig. 1. A realization of the electric field at a position $\boldsymbol{\rho}$ is written as a column vector $\mathbf{E}(\boldsymbol{\rho},\omega) = [E_x(\boldsymbol{\rho},\omega), E_y(\boldsymbol{\rho},\omega), E_z(\boldsymbol{\rho},\omega)]^{\mathrm{T}}$, where T denotes the matrix transpose and ω is the angular frequency. The coherence properties of the source are described by the cross-spectral density matrix (CSDM) defined as

$$\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \langle \mathbf{E}^*(\boldsymbol{\rho}_1,\omega)\mathbf{E}^{\mathrm{T}}(\boldsymbol{\rho}_2,\omega)\rangle, \tag{1}$$

where the angle brackets and the asterisk stand for the ensemble average and complex conjugation, respectively. The CSDM elements are explicitly given by

$$\begin{aligned} W_{\alpha\beta}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) &= \langle E_\alpha^*(\boldsymbol{\rho}_1,\omega)E_\beta(\boldsymbol{\rho}_2,\omega)\rangle,\\ &= [S_\alpha(\boldsymbol{\rho}_1,\omega)]^{1/2}[S_\beta(\boldsymbol{\rho}_2,\omega)]^{1/2}\mu_{\alpha\beta}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega),\\ &\quad (\alpha,\beta)\in(x,y,z). \end{aligned} \tag{2}$$

Above, $S_\alpha(\boldsymbol{\rho},\omega)$ is the spectral density of $E_\alpha(\boldsymbol{\rho},\omega)$ and $\mu_{\alpha\beta}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$ is the complex correlation coefficient between $E_\alpha(\boldsymbol{\rho}_1,\omega)$ and $E_\beta(\boldsymbol{\rho}_2,\omega)$. Setting $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$ in Eq. (1) yields the (spectral) polarization matrix $\boldsymbol{\Phi}(\boldsymbol{\rho},\omega) = \mathbf{W}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)$ which characterizes the state of polarization of the source. The spatial-frequency components of the electric field are obtained by the Fourier transform

$$\tilde{\mathbf{E}}(\boldsymbol{\kappa},\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\mathbf{E}(\boldsymbol{\rho},\omega)\exp(-i\boldsymbol{\kappa}\cdot\boldsymbol{\rho})\mathrm{d}^2\boldsymbol{\rho}, \tag{3}$$

and their correlations are represented by the angular correlation matrix (ACM), expressed as [17]

$$\begin{aligned} \mathbf{T}(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\omega) &= \langle \tilde{\mathbf{E}}^*(\boldsymbol{\kappa}_1,\omega)\tilde{\mathbf{E}}^{\mathrm{T}}(\boldsymbol{\kappa}_2,\omega)\rangle, &(4)\\ &= \frac{1}{(2\pi)^4}\iint_{-\infty}^{\infty}\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\exp[i(\boldsymbol{\kappa}_1\cdot\boldsymbol{\rho}_1-\boldsymbol{\kappa}_2\cdot\boldsymbol{\rho}_2)]\mathrm{d}^2\boldsymbol{\rho}_1\mathrm{d}^2\boldsymbol{\rho}_2. &(5) \end{aligned}$$

We note that $\mathbf{T}(\boldsymbol{\kappa}_1,-\boldsymbol{\kappa}_2,\omega)$ constitutes the Fourier transform of $\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$. Furthermore, in terms of the ACM the far-field CSDM can be written as [16,17]

$$\mathbf{W}^{(\infty)}(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega) = (2\pi k)^2\cos\theta_1\cos\theta_2\mathbf{T}(k\boldsymbol{\sigma}_1,k\boldsymbol{\sigma}_2,\omega)\frac{\exp[ik(r_2-r_1)]}{r_1r_2}, \tag{6}$$

where (see Fig. 1) $r_i = |\mathbf{r}_i|$ and $\hat{\mathbf{s}}_i = \mathbf{r}_i/r_i$, with $\mathbf{r}_i$ being the observation point. In addition, k is the wave number, θ_i is the angle between $\hat{\mathbf{s}}_i$ and the z axis, and $\boldsymbol{\sigma}_i$ is the transverse component of $\hat{\mathbf{s}}_i$, $i\in(1,2)$.

The field far from the source is locally planar in all directions and thus we may express the CSDM in spherical polar coordinates as a 3×3 matrix with only four nonzero elements (no radial component in any direction). The transformation from Cartesian coordinates to spherical polar coordinates is done by projecting the field to the local unit vectors which are specified by the directions $\hat{\mathbf{s}}_i$, $i\in(1,2)$. With this procedure the far-field CSDM in spherical polar coordinates takes the form

$$\mathcal{W}^{(\infty)}(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega) = \mathbf{U}^{\mathrm{T}}(\hat{\mathbf{s}}_1)\mathbf{W}^{(\infty)}(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega)\mathbf{U}(\hat{\mathbf{s}}_2), \tag{7}$$

where $\mathbf{U}(\hat{\mathbf{s}}_i) = [\hat{\mathbf{u}}_r(\hat{\mathbf{s}}_i),\hat{\mathbf{u}}_\theta(\hat{\mathbf{s}}_i),\hat{\mathbf{u}}_\varphi(\hat{\mathbf{s}}_i)]$ is a 3×3 transformation matrix, and

$$\hat{\mathbf{u}}_r(\hat{\mathbf{s}}_i) = [\sin\theta_i\cos\varphi_i, \sin\theta_i\sin\varphi_i, \cos\theta_i]^{\mathrm{T}}, \tag{8}$$

$$\hat{\mathbf{u}}_\theta(\hat{\mathbf{s}}_i) = [\cos\theta_i\cos\varphi_i, \cos\theta_i\sin\varphi_i, -\sin\theta_i]^{\mathrm{T}}, \tag{9}$$

$$\hat{\mathbf{u}}_\varphi(\hat{\mathbf{s}}_i) = [-\sin\varphi_i, \cos\varphi_i, 0]^{\mathrm{T}}, \tag{10}$$

are the unit vectors of the spherical polar coordinates. The matrix $\mathbf{U}(\hat{\mathbf{s}})$ is an orthogonal matrix satisfying $\mathbf{U}(\hat{\mathbf{s}})\mathbf{U}^{\mathrm{T}}(\hat{\mathbf{s}}) = \mathbf{U}^{\mathrm{T}}(\hat{\mathbf{s}})\mathbf{U}(\hat{\mathbf{s}}) = \mathbf{I}$, where $\mathbf{I}$ is the 3×3 unit matrix. Since the far field is transverse, i.e., $[\hat{\mathbf{u}}_r(\hat{\mathbf{s}})]^{\mathrm{T}}\mathbf{E}(r\hat{\mathbf{s}},\omega) = 0$, the elements in the first row and first column of $\mathcal{W}^{(\infty)}(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega)$ are zero. Inserting Eq. (6) into Eq. (7), we find that

$$\mathcal{W}^{(\infty)}(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega) = a(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega)\mathbf{U}^{\mathrm{T}}(\hat{\mathbf{s}}_1)\mathbf{T}(k\boldsymbol{\sigma}_1,k\boldsymbol{\sigma}_2,\omega)\mathbf{U}(\hat{\mathbf{s}}_2), \tag{11}$$

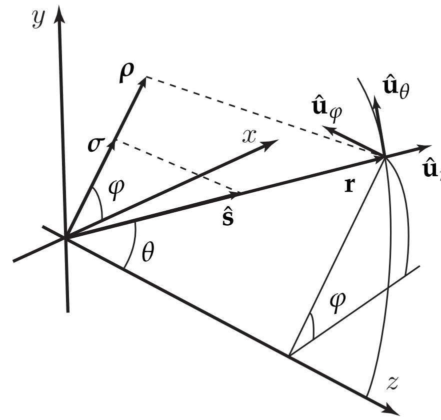

Fig. 1. Illustration of the geometry and notations pertaining to a finite planar source at the plane $z = 0$. The vector $\mathbf{r}$ specifies the far-field observation point while $\hat{\mathbf{s}}$ is the related (unit) direction vector. The transverse components of $\mathbf{r}$ and $\hat{\mathbf{s}}$ are denoted by $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}$, respectively. The vectors $\hat{\mathbf{u}}_r$, $\hat{\mathbf{u}}_\theta$ and $\hat{\mathbf{u}}_\varphi$ are the unit vectors associated with the spherical polar coordinates (r,φ,θ).

with $a(r_1\hat{\mathbf{s}}_1,r_2\hat{\mathbf{s}}_2,\omega) = (2\pi k)^2\cos\theta_1\cos\theta_2\exp[ik(r_2-r_1)]/r_1r_2$. Eq. (11) indicates that measuring the far-zone CSDM for all pairs of directions $\mathbf{s}_i$ yields the complete low spatial-frequency part of the ACM, i.e., $\mathbf{T}(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\omega)$ with $|\boldsymbol{\kappa}_i|\le k$, $i\in(1,2)$. The high spatial-frequency components with $|\boldsymbol{\kappa}_i| > k$ correspond to evanescent waves and their contributions are negligible if the spatial field and correlation variations of the source take place on scales much longer than the wavelength of the light.

3. Quasihomogeneous source

Next we assume that the source is quasihomogeneous, i.e., the correlation (coherence) lengths between all pairs of field components (parallel and orthogonal) are, at all frequencies, short compared to the length scale of spectral density variations [8]. In addition, we assume that the source dimensions are much larger than the wavelengths present in the spectrum and the various correlation lengths. These conditions are met for many sources with practical importance and lead to reduced complexity between the far-field and source information.

The CSDM of an electromagnetic quasihomogeneous source can be written in a matrix form as [12,18]

$$\mathbf{W}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \mathbf{S}^{1/2}(\boldsymbol{\rho}',\omega)\,\boldsymbol{\mu}(\Delta\boldsymbol{\rho},\omega)\,\mathbf{S}^{1/2}(\boldsymbol{\rho}',\omega), \tag{12}$$

where $\boldsymbol{\rho}' = (\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)/2$ and $\Delta\boldsymbol{\rho} = \boldsymbol{\rho}_2-\boldsymbol{\rho}_1$ are the average and difference position vectors, respectively, and the superscript $1/2$ denotes the (positive) square root. The matrix $\mathbf{S}(\boldsymbol{\rho}',\omega) = \mathrm{diag}[S_x(\boldsymbol{\rho}',\omega), S_y(\boldsymbol{\rho}',\omega), S_z(\boldsymbol{\rho}',\omega)]$ is diagonal and composed of the spectral densities of the field components, while the elements of $\boldsymbol{\mu}(\Delta\boldsymbol{\rho},\omega)$ are the correlation coefficients $\mu_{\alpha\beta}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \mu_{\alpha\beta}(\Delta\boldsymbol{\rho},\omega)$, with $(\alpha,\beta)\in(x,y,z)$. Since the source is quasihomogeneous, the elements of $\mathbf{S}(\boldsymbol{\rho}',\omega)$ vary much more slowly with $\boldsymbol{\rho}'$ than the elements of $\boldsymbol{\mu}(\Delta\boldsymbol{\rho},\omega)$ change with $\Delta\boldsymbol{\rho}$. The source CSDM in Eq. (12) is consistent with the quasihomogeneous beam-field sources considered by many authors [9–13]. In addition, an explicit example of a nonparaxial source field described by Eq. (12) is blackbody radiation in an aperture of a large cavity [14,15].

By inserting Eq. (12) into Eq. (5) and changing the integration variables from $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ to $\boldsymbol{\rho}'$ and $\Delta\boldsymbol{\rho}$, the ACM of a quasihomogeneous source can be written as

$$\mathbf{T}(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\omega) = \tilde{\mathbf{Q}}(\Delta\boldsymbol{\kappa},\omega)\circ\tilde{\boldsymbol{\mu}}(\boldsymbol{\kappa}',\omega), \tag{13}$$

where $\circ$ denotes the element-wise product, $\Delta\boldsymbol{\kappa} = \boldsymbol{\kappa}_2-\boldsymbol{\kappa}_1$, and $\boldsymbol{\kappa}' = (\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)/2$. The elements of $\tilde{\mathbf{Q}}(\Delta\boldsymbol{\kappa},\omega)$ and the matrix $\tilde{\boldsymbol{\mu}}(\boldsymbol{\kappa}',\omega)$ are given