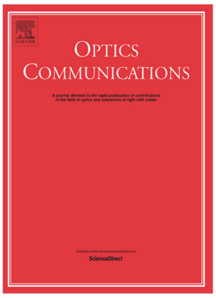

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Multiple photon effective Hamiltonians in linear quantum optical networks

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Abstract

We give an alternative derivation for the explicit formula of the effective Hamiltonian describing the evolution of the quantum state of any number of photons entering a linear optics multiport. The description is based on the effective Hamiltonian of the optical system for a single photon and comes from relating the evolution in the Lie group that describes the unitary evolution matrices in the Hilbert space of the photon states to the evolution in the Lie algebra of the Hamiltonians for one and multiple photons. Our derivation complements previous results with a point of view which reminds of the Schrödinger picture. These group theory results allow us to prove that quantum optical linear networks obey different proportionality rules which relate the expected photon number at each input and output ports. In particular, we show that for a uniform photon input the average photon number at the output is preserved for any linear multiport.

Keywords: Quantum optics, Linear optics multiports, Group theory, Lie groups, Lie algebras
PACS: 42.50.-p, 42.79.-e, 02.20.-a, 02.20.Sv

1. Introduction: Group theory and quantum optics

1.1. Linear quantum-optical networks

The evolution of the quantum states of light when they pass linear optical networks can be described from the classical scattering matrix of the network S. In classical electromagnetism, S relates the amplitudes of the input fields in m input modes with the amplitudes of the m output modes and has many applications in microwave circuit design [1].

In quantum optics, we can replace the field amplitudes with the probability amplitudes in the wavefunction of a photon and use S to see the evolution of the creation operator in each mode.

However, when there are multiple photons, the evolution does not only include wave interference effects, like in classical electromagnetic waves, but also purely quantum effects related to the bosonic nature of the photons. A most striking example is the Hong-Ou-Mandel effect in which two independent photons that reach simultaneously the two separate inputs of a beam splitter always come out together [2]. These interactions have no classical counterpart and are behind the ability of linear optical systems to give an efficient solution to the boson sampling problem, a task that is strongly believed to be inefficient in any classical computing machine [3].

In this paper, we describe the quantum evolution of photons through linear optical elements from results from group theory. Working with Lie algebras, we show different proportionality rules for the expected photon number at each output port of a linear optical network.

We study photonic states

$$\bigotimes_{k=1}^{m} |n_k\rangle_k = |n_1\rangle_1 |n_2\rangle_2 \dots |n_m\rangle_m = |n_1 n_2 \dots n_m\rangle \tag{1}$$

with n photons that are distributed into m orthogonal modes. In the most general case, these modes represent any set of orthogonal single photon states so that

$$_k\langle 1|1\rangle_l = \delta_{k,l}. \tag{2}$$

The modes can be different paths, which gives a very intuitive picture of the network, but they can also represent orthogonal temporal wavefunctions, different directions in the same spatial path, photons in orthogonal polarization states, photons with different orbital angular momentum or with a different frequency.

We consider linear optical systems where the number of photons is conserved

$$\sum_{k=1}^{m} n_k = n. \tag{3}$$

In passive lossless systems the total energy is conserved, which fits well with the description in terms of classical fields. The quantum equivalent is a conservation of probability. If we have a superposition of n photon states, the output will be a different superposition where all the photon states must sum to a probability of one. The input and output states are related by a unitary operator

$$|\psi\rangle_{out} = U |\psi\rangle_{in}. \tag{4}$$

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