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Generation of stable and accurate solitary waves in a viscous numerical wave tank

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ABSTRACT

The propagation of solitary waves in a constant water depth is investigated. The Dirichlet boundary condition and an internal mass source are utilized, respectively, to generate the desired solitary waves. Various solitary wave theories are applied to the numerical model. The goal is to generate stable and accurate solitary waves. Accuracy is evaluated in terms of the relative error of the wave height between the input signal and the generated wave and stability is evaluated in terms of the distance required to stabilize the waves. The attenuation of solitary waves propagating over a significant travel distance due to the viscous effect is then studied experimentally and numerically. The results reveal that the use of the first-order solitary wave solution with the Dirichlet boundary condition is surprisingly good while the use of the ninth-order solitary wave solution with the internal mass source provides the best performance. It is conjectured that in the numerical implementation, the use of internal wavemaker acquires less theoretical information of solitary wave properties than that of the Dirichlet boundary condition such that the former approach demands a higher-order solitary wave solution to generate accurate and stable waves.

1. Introduction

One of the key points in the modeling of wave-structure interactions is the accurate generation of the desired waves. If the desired waves cannot be generated correctly, the results cannot be modeled with confidence. It is thus of great importance to verify the stability and accuracy of generated waves prior to the formal testing of wave-structure interaction. As a wave propagates into a shallow-water region, an isolated solitary wave and cnoidal waves are frequently utilized to model the nearshore processes of wave-structure interactions (e.g., Wu and Hsiao, 2013, 2017; Wu et al., 2012, 2014b) and wave transformation (e.g., Chang and Lin, 2015; Hsiao et al., 2008; Lin et al., 2014; Wu et al., 2015, 2018). Moreover, a solitary wave with only a crest (i.e., without trough) can be easily generated either in physical modeling or in numerical simulation, providing an idealized investigation without the effects of preceding or following waves.

Since the discovery by John Russell (1845) of solitary waves in eighteenth century, which feature a stable waveform and negligible wave damping over a significantly long travel distance, many theoretical studies have been devoted to the approximation of the wave properties of solitary waves in terms of waveform, wave celerity and particle velocities. Among existing theories of solitary waves, the Boussinseq theory of solitary waves (Boussinesq, 1871) has been most commonly used even though the solution has first-order accuracy. Experimentally, Goring (1978) approximated the paddle movement of a wavemaker based on the theory of Boussinesq (1871). A wave generation algorithm was applied to a numerical wave tank with a virtual wave paddle by Huang and Dong (2001). Wu et al. (2014a) numerically examined the method of Goring (1978) implemented with various solitary wave theories for wave generation using a mesh-free method to solve the fully non-linear potential flow equations. This concept has been successfully implemented for a two-dimensional (2D) wave flume with a piston-type wavemaker (Wu et al., 2016). Due to the neglect of the viscous effect, solitary waves do not experience any damping during propagation, which is not realistic. It has long been noted that a solitary wave would be damped as it propagates over a considerable distance due to the viscous and frictional effects of the seafloor (Keulegan, 1948; Mei, 1983). Motived by Wu et al. (2014a), Farhadi et al. (2016) investigated the accurate generation of solitary waves using various paddle motions of a piston-type wavemaker implemented using an incompressible smoothed particle hydrodynamics (SPH) model. Furthermore, the model results of Wu et al. (2014a) and Farhadi et al. (2016) are consistent in showing that the use of the ninth-order solution of solitary waves provides the best performance.

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To generate water waves in a numerical wave tank, in addition to using a virtual wave paddle, two other approaches can be used, namely the Dirichlet boundary condition and the internal wavemaker. The former is just to implement the appropriate theory of any surface gravity wave in terms of free surface elevation and particle velocities within the whole water column at the inflow boundary, and the latter involves putting an internal source inside the computational domain, with the source balanced by either mass or momentum conservations.

The present study focuses on how to generate stable and accurate solitary waves using the Dirichlet boundary condition and an internal mass source, respectively, with various existing theories of solitary waves. Accuracy is measured in terms of the wave height variation between the input signal to the model and the generated wave, which should be as small as possible, and the stability is judged in terms of the viscous damping of solitary waves between the numerical results and the existing theories. It should be noted that the input wave height for the model may not be completely equivalent to the generated wave height, and that there may be some variation between them due to the numerical instability or inaccuracy of theoretical results. Seiffert et al. (2014) claimed that applying different solitary wave theories to the wavemaker do not produce any significant differences, and that once a wave completely enters the fluid domain, it is modified. Although we agree with the latter viewpoint, the former statement has been challenged by Wu et al. (2014a) and thus further investigation is required. There is no doubt that the wave properties will change when a wave enters a nonlinear system either in a physical wave flume or in a numerical wave tank based on the Navier-Stokes equations. However, the issue of how and when a solitary wave will reach its stable state when different solitary wave theories are used is still unsolved. It would be valuable to provide guidelines for numerical modelers on how much distance is required for a solitary wave to become stable for a given wave height for a certain solitary wave theory. To better understand the behavior of solitary wave damping over a significant travel distance, experiments were performed to confirm the viscous damping of solitary waves and compare the results with theoretical solutions. Particle image velocimetry (PIV) was used to measure the particle velocities of solitary waves. The velocity fields obtained from using PIV were used to understand the wave properties of solitary waves propagating over a long distance for the wavemaker movement prescribed by using the first-order solution of solitary waves. Model-data comparisons are made in terms of the free surface elevation and velocity time series.

The rest of this paper is organized as follows. After a brief description of the issue of concern, a summary of existing solitary wave theories is given in Section 2. Basic information regarding the numerical model is given in Section 3.1, including model equations, initial and boundary conditions, and two approaches for generating the desired solitary waves. Laboratory-scale measurements are described in Section 3.2. Section 4 presents the relative error between the input signal and the generated wave amplitude to verify the stability and accuracy of the generated solitary waves. Measured data are utilized to support the numerical calculations. Finally, conclusions are presented in Section 5.

2. Overview of existing solitary wave theories

The simplest form of a solitary wave is the Boussinesq theory of solitary waves (Boussinesq, 1871), in which the leading-order solitary wave, propagating at a constant depth, h, can be expressed as:

$$\eta(x,\, t) = H \mathrm{sech}^2[K(x - ct)], \tag{1}$$

where $\eta(x, t)$ represents the free surface elevation, H is the wave height, $K = \sqrt{3H/4h^3}$ is the effective wave number and $c = \sqrt{g(h + H)}$ is the wave celerity, with the x-coordinate pointing in the direction of wave propagation and t representing time.

McCowan (1891) proposed a theoretical solution for the wave properties of solitary waves, where the mathematical formulations can be found in Lee et al. (1982). We note that the wave properties of using McCowan's solitary wave theory cannot be directly calculated because it features an implicit formulation, and thus needs to be solved iteratively using a method such as the Newton-Raphson method. Grimshaw (1971) proposed a solitary wave theory derived from a series expansion up to the third-order accuracy based on the assumption that the fluid is incompressible, irrotational, and inviscid, i.e., based on the Euler equations. Fenton (1972) derived a solitary wave solution in the form of a series expansion up to the ninth-order accuracy based on the Euler equations. Some coefficients in the formulations were determined numerically in order to make the ninth-order solution possible. Of note, only the free surface displacement and wave celerity were provided by Fenton (1972) but the particle velocities was not available. The solution of a solitary wave by Fenton (1972) up to the third-order accuracy is identical to that of Grimshaw (1971).

Some higher-order solutions of solitary waves have been reported, which are, in principle, more precise than the ninth-order solution. For example, Wu et al. (2005) derived a solution of solitary waves up to the eighteenth-order accuracy; however, a large number of coefficients are involved in the formulations. Some studies have provided exact solutions of solitary waves based on the Euler equations, for example those of Tanaka (1986) and Dutykh and Clamond (2014). To obtain these solitary wave solutions, numerical algorithms were used to solve the governing equations and then numerical iterations were required. Thus, there is no general formulations for the waveform or particle velocity.

In the present study, direct comparisons in terms of free surface displacements and particle velocities at the free surface in time histories are performed among the theories of Boussinesq (1871), McCowan (1891), Grimshaw (1971), Fenton (1972), Tanaka (1986), and Dutykh and Clamond (2014). Two wave conditions are demonstrated, for the cases of $H/h = 0.20$ and 0.40, in a constant water depth of $h = 40$ cm, as shown in Figs. 1 and 2, respectively.

For the case with $H/h = 0.20$ and $h = 40$ cm, all solitary wave solutions are mostly identical, except for the ones obtained from the theories of Boussinesq (1871) and McCowan (1891). The free surface time series of Boussinesq (1871) exhibits the narrowest outskirt of the wave, and this may be partly due to the effective wave number, K, being larger than those for the other theories. For the particle velocities at the still water level, the solutions are typically unique for each solitary wave theory. The numerically calculated exact solutions of Tanaka (1986) and Dutykh and Clamond (2014) are the same. The maximum horizontal velocity at the still water level obtained from the Boussinesq (1871) is comparable to the exact solution of Tanaka (1986); that calculated using the third-order solution of Grimshaw (1971) exhibits the largest underestimation, followed by McCowan's theory. Although the particle velocity of the ninth-order theory (Fenton, 1972) is unavailable, we compare the wave celerity for this theory with those obtained from the numerical solutions of Tanaka (1986) and Dutykh and Clamond (2014). It is found that these three solutions are the same up to seven digits of accuracy.

Considering the case with $H/h = 0.40$ and $h = 40$ cm, apparent variation is observed for the free surface time series. The free surface displacements and wave celerity obtained from the two exact numerical solutions and the ninth-order theory are identical. This result is consistent with the study of Dutykh and Clamond (2014), who pointed out that for $H/h \leq 0.50$, the ninth-order solution is comparable to the exact numerical solution. The Boussinesq theory provides the narrowest outskirt boundary near the still water level due to the relatively larger effective wave number, while the McCowan's theory exhibits the narrowest waveform near the crest. The solution of Grimshaw (1971) is very close to the exact solutions as well as Fenton's solution. Once again, the particle velocities at the still water level show significant variations among the theories. The velocities obtained from the exact solution exhibit the highest magnitude, followed by those obtained using the Boussinesq theory and McCowan's solution. Grimshaw's theory provides the lowest particle velocities.