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Solving coupled beam-fluid interaction by DTM

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ABSTRACT

This paper aims to employ the differential transformation method for solving a fluid-structure interaction problem. The free vibration of an Euler-Bernoulli beam next to an incompressible, irrotational and inviscid fluid is administrated by a coupled boundary value problem. In the present study, the governing differential equations are solved by the mentioned semi-analytical approach. To evaluate the correctness and robustness of the developed formulations, the free vibration of several beam-fluid systems, with different boundary conditions, are assessed. Moreover, the obtained results are compared with those of the finite element technique.

1. Introduction

In offshore structures, dam-reservoir systems, aircraft wings and long-span bridges in contact with fluid, the beam like-structures interacting with fluid can be observed. Note that; a coupled boundary value problem governs the vibrational behavior of the flexible beam-fluid systems. Obviously, investigating the dynamic behavior of these systems is essential for reliable design and life prediction. Various researchers have dealt with the dynamic behavior of these systems. In what follows, the corresponding state of the art are briefly reviewed.

In 1970, **Jones** experimentally and analytically assessed the vibration of beams immersed in a fluid and carrying concentrated mass and rotary inertia (Jones, 1970). Afterwards, **Taleb** and **Misra** analyzed axially moving slender beam immersed in an incompressible fluid (Taleb and Misra, 1980). In this work, the transverse displacement of the beam was written in terms of time-dependent admissible functions. Subsequently, the set of non-autonomous ordinary differential governing equations were derived. Moreover, **Nagaya** suggested a method for solving problems of transient response in flexure of beams with concentrated tip inertias (Nagaya, 1985). In this study, the beam was immersed in a fluid, and it had a variable cross-section. Then, **Chang** and **Liu** dealt with the natural frequencies of the immersed restrained columns (Chang and Liu, 1989).

In 1997, by using the separation of variables' approach, the natural vibration of a flexible beam-water system was analytically assessed by Xing et al. (1997). These researchers assumed that the coupled systems were subjected to an undisturbed boundary condition at infinity of the water domain and a linear surface disturbance condition on the free surface. Sader (1998) presented the frequency response of a cantilever beam immersed in a viscous fluid and excited by arbitrary thermal driving force (Sader, 1998). In another research, the theoretical model of Ref (Sader, 1998) was experimentally validated by Chon et al. (2000).

Zhao et al. took advantage of the separation of variables' scheme for computing the natural frequencies of a flexible beam-water coupled system with a concentrated mass attached at its free end (Zhao et al., 2002). Besides, **Green** and **Sader** dealt with the torsional frequency response of cantilever beams immersed in viscous fluids (Green and Sader, 2002). By taking into account the added mass concept, a closed-form solution for computing the natural frequencies of the immersed beams was proposed by Wu and Chen (2003). These researchers compared the results obtained from their strategy with those of the finite element tactic. In 2006, **Wu** and **Hsu** presented a unified algorithm to approximate the several lowest natural frequencies and the corresponding mode shapes of an elastically supported immersed uniform beam carrying an eccentric tip mass with rotary inertia (Wu and Hsu, 2006). In this method, the distributed added mass along the immersed part of the beam was modeled with a number of concentrated added mass. For verification, these researchers compared the results of their scheme with those of the finite element approach.

Among this kind of the research, **Gorman** et al. analytically computed the natural frequencies of the cantilever beam in contact with a fluid cavity (Gorman et al., 2007). At the next stage, by using the obtained values, they estimated the natural frequencies of the system in the absence of the fluid interaction. In two-dimensional plane, Basak and Raman (2007) analyzed the long and slender, micromechanical beam oscillating in an incompressible viscous fluid by utilizing the boundary integral technique (Basak and Raman, 2007). With the help of separation of variables, **Xing** investigated the dynamic behavior of planar flexible

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Notation

A	Cross-sectional area of the beam
$\mathbf{B}$	Interaction matrix
$\mathbf{B}_e$	Elemental interaction matrix
$\mathbf{D}$	Nodal displacement vector
EI	Flexural rigidity of the beam
${}_1F_2$	Hypergeometric function
F[k], U[k], Z[k]	The differential transformation function
H	Beam length
$\mathbf{H}$	Generalized fluid stiffness matrix
$\mathbf{H}_e$	Generalized elemental fluid stiffness matrix
$\mathbf{K}$	Stiffness matrix of the structure
$\mathbf{K}_e$	Elemental stiffness matrix of the structure
L	Length of the fluid domain
L_e	Element length
$\mathbf{M}$	Mass matrix of the structure
$\mathbf{M}_a$	Added mass matrix
$\mathbf{M}_e$	Elemental mass matrix of the structure
M, N	Number of series terms
$\mathbf{P}$	Nodal pressure vector
P, P^*	Pressure function
$b_k^j, c_j, d_j, e_k^j, g_k^j$	Parameter defined in section 4
$f(x)$	An arbitrary function
j, k, q	Series numerator
x, y	Cartesian coordinates
u	Beam deflection function
$z_j(\xi)$	Function defined in Eq. (13)
Λ_j	Parameter defined in Eq. (16)
η, ξ	Dimensionless coordinates
λ_j	Parameter defined in Eq. (14)
ρ	Beam density
ρ_f	Fluid density
ω	Natural frequency of the system

slender structure-water interaction systems (Xing, 2007). In this work, the **Sommerfeld** radiation condition was applied at the infinity of the water domain. By employing the **Runge-Kutta** algorithm, **Gosselin** et al. solved the equations of motions of a flexible slender cantilever beam extending axially in a horizontal plane at a known rate while it was immersed in an incompressible fluid (Gosselin et al., 2007).

In another event, **Jin** and **Xing** took advantage of a mixed mode function-boundary element method to solve the transient dynamics of a floating beam-water interaction system excited by the impact of a landing beam (Jin and Xing, 2007). Moreover, **Weiss** et al. calculated the hydrodynamic resistance functions, which described the relationship between the resistance coefficients, the fluid's density and the vibration characteristics of an immersed beam (Weiss et al., 2008). For this purpose, a semi-numerical method was suggested for solving the associated fluid-structure interaction problem. By employing the differential quadrature approach, **Lin** and **Qiao** computed the natural frequencies of an axially moving beam in fluid (Lin and Qiao, 2008). Besides, **Rezazadeh** et al. investigated the flexural vibrations of an electrostatically actuated cantilever micro-beam in an incompressible inviscid stationary fluid, based on the three-dimensional aerodynamic theory (Rezazadeh et al., 2009).

In 2011, **Miquel** and **Bouaanani** proposed a practical procedure for dynamic analysis of beams laterally vibrating in contact with water on one or both sides (Miquel and Bouaanani, 2011). In this technique, structure flexibility, soil flexibility, varying water levels and various boundary condition were taken into account. **Keyvani** et al. developed a closed-form formulation for dynamic analysis of a shear beam-compressible fluid system (Keyvani et al., 2013). Furthermore, they compared the obtained results with those of the finite element scheme. Based on **Fourier-Bessel** series expansions and linear potential theory, **Shabani** et al. developed an analytical tactic for finding the solution of the eigenvalue problem for a cantilever micro-beam submerged in a bounded incompressible fluid domain (Shabani et al., 2013). In another study, **Sharafkhani** et al. studied the transient response of electrostatically excited micro-beam interacting with bounded compressible fluid by applying **Fourier-Bessel** series expansions (Sharafkhani et al., 2013).

In 2014, **Eftekhari** and **Jafari** suggested a simple mixed modal-differential quadrature strategy for vibration of beams in contact with fluid (Eftekhari and Jafari, 2014). These researchers studied both the free and forced vibration of this system. With the help of **Galerkin** scheme, **Ni** et al. evaluated the free vibration and stability of the cantilever beam attached to an axially moving base in fluid (Ni et al., 2014). Moreover, **Jafari-Talookolaei** and **Lasemi-Imani** presented the free vibration characteristics of a laminated composite beam partially contacting with a fluid by developing an analytical method (Jafari-Talookolaei and Lasemi-Imani, 2015). Besides, **Bouaanani** and **Miquel** proposed a simplified procedure for finding the modal dynamic and earthquake response of the coupled flexible beam-fluid systems (Bouaanani and Miquel, 2015). In this study, it was assumed that the beam had lateral interaction with one or two semi-infinite fluid domain.

It is worthwhile to highlight that an effective scheme for finding the solution of linear, and nonlinear boundary, initial and eigenvalue problems is the differential transformation method (DTM). This semi-analytical technique is based on **Taylor** series expansion (Abdel-Halim Hassan, 2008; Chen and Ho, 1996; Jang et al., 2000). Up to now, many researchers took advantage of DTM for solving the various eigenvalue problems. **Malik** and **Dang** employed the differential transformation algorithm for calculating the eigen-pairs of thin beams (Malik and Huy Dang, 1998). Afterwards, **Chen** and **Ho** solved the transverse vibration of rotating **Timoshenko** beams under axial loading by DTM (Chen and Ho, 1999). In 2000, **Hasan** used DTM for eigenvalues and normalized eigen-functions for a **Sturm–Liouville** eigenvalue problem (Abdel-Halim Hassan, 2002). Then, **Zeng** and **Bert** used the differential transformation approach for vibration analysis of a tapered bar (Zeng and Bert, 2001). In another study, these researchers deployed this semi-analytical method to find the solution of the compound bars (Bert and Zeng, 2004). Furthermore, **Yeh** et al. compared the ability of the finite difference tactic and DTM in analyzing the free vibration of plates (Yeh et al., 2006). Furthermore, **Catal** applied the differential transformation scheme in solving the free vibration equations of the beam on elastic soil (Çatal, 2008). In 2009, **Chen** et al. took advantage of DTM for extracting the natural frequencies and mode shapes of marine risers (Chen et al., 2009). Additionally, DTM was utilized for analyzing the out-of-plane free vibration of rotating the tapered beams in the post-elastic regime by Das et al. (2009). Besides, Yalcin et al. (2009) dealt with the free vibration analysis of circular plates, based on DTM (Yalcin et al., 2009). Analogously, Lal and Ahlawat (2015) analyzed the free vibration of a functionally graded circular plate (Lal and Ahlawat, 2015).

To authors' best knowledge, DTM has not been previously employed in free vibration analysis of the beam-fluid system, although it has been applied in different eigenvalue problems. Consequently, this paper aims to find the free vibration solution of a beam vibrating in contact with a finite fluid domain by taking advantage of the differential transformation approach. In this process, various boundary conditions are considered for the beam. Besides these activities, proper comparison studies will be performed, and the related discussions will be given throughout the article.