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Unravelling cosmological perturbations

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ABSTRACT

We explain in detail the quantum-to-classical transition for the cosmological perturbations using only the standard rules of quantum mechanics: the Schrödinger equation and Born's rule applied to a subsystem. We show that the conditioned, i.e. intrinsic, pure state of the perturbations, is driven by the interactions with a generic environment, to become increasingly localized in field space as a mode exists the horizon during inflation. With a favourable coupling to the environment, the conditioned state of the perturbations becomes highly localized in field space due to the expansion of spacetime by a factor of roughly $\exp(-c\Delta N)$, where $\Delta N \sim 50$ and c is a model dependent number of order 1. Effectively the state rapidly becomes specified completely by a point in phase space and an effective, classical, stochastic process emerges described by a classical Langevin equation. The statistics of the stochastic process is described by the solution of the master equation that describes the perturbations coupled to the environment.

1. Introduction

It is breathtaking that quantum fluctuations [1–6] in the inflating universe [7–12] became the seeds of the structure in the universe and were imprinted as small fluctuations of the CMB. However, there seems to be a missing chapter in the standard story, namely, how the quantum fluctuations actually became classical. If one is only interested in probability distributions then the issue can be discussed only in terms of the degree of decoherence and so it sits there as the elephant in the room.

The goal here is to go beyond a description of a quantum process in terms of probabilities to one that can describe the trajectory of a single system. The classic "Wigner's Friend" thought experiment illustrates the issues involved in a very simple setting, but one that is not meant to be at all realistic.[1] The friend measures a qubit initially in the state $c_+|+\rangle + c_-|-\rangle$ and according to the external observer, Wigner, the total state of the system is the entangled state $c_+|+\rangle|F_+\rangle + c_-|-\rangle|F_-\rangle$. More specifically, Wigner associates the reduced state $\rho = |c_+|^2|F_+\rangle\langle F_+| + |c_-|^2|F_-\rangle\langle F_-|$ to the friend. On the contrary, for the friend, Born's rule implies that their state is either $|\psi\rangle = |F_\pm\rangle$ with probability $|c_\pm|^2$, respectively.[2] So the state of the system depends on the frame of reference. The external observer, Wigner, describes the friend with the *unconditioned* state ρ whereas in the friend's frame their state $|\psi\rangle$ is *conditioned*:

W (unconditioned state):

$$\rho = |c_+|^2|F_+\rangle\langle F_+| + |c_-|^2|F_-\rangle\langle F_-| \, ,$$

F (conditioned state):

$$|\psi\rangle = \begin{cases} |F_+\rangle & \text{prob} = |c_+|^2 \, , \\ |F_-\rangle & \text{prob} = |c_-|^2 \, . \end{cases} \tag{1.1}$$

The two states are related via a stochastic average

$$\rho = \mathscr{E}(|\psi\rangle\langle\psi|) \, . \tag{1.2}$$

The key distinction between the two states is that the unconditioned state ρ exhibits entanglement – it is a mixed state – while the conditioned state $|\psi\rangle$ is pure but random. So there is a *duality of perspective* between entanglement and randomness: $\rho \longleftrightarrow |\psi\rangle$, which gives rise to a form of *observer complementarity*.[3]

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[1] True macroscopic systems cannot be described by simple states like $|F_\pm\rangle$ because of entanglement with the environment that is being ignored here. In the simple toy model, it is the qubit that effectively decoheres the friend.

[2] The states $|F_\pm\rangle$ are the states in the generically unique decomposition of ρ into an orthogonal ensemble or, equivalently, the eigenvectors of ρ.

[3] By recognizing that the state depends on the frame of reference (or perspective, or context) one realizes a unification of *many worlds* and *Copenhagen* quantum mechanics.

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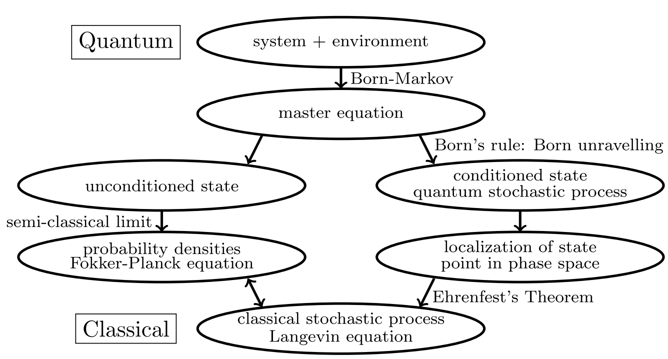
In the context of the cosmological perturbations, the unconditioned state ρ is the one that is analysed in concrete models involving their interaction with some environment, consisting either of other fields or self interactions of the perturbations. After various approximations, the state ρ satisfies a *master equation* that describes how the perturbations are decohered by the environment. This points to the fact that a classical description should be valid and probability densities can then be extracted from ρ. However, if one wants to describe how a single perturbation becomes classical, we need a description of the trajectory of the individual mode, in other words the state from the frame of the reference of the mode itself. This is the conditioned state constructed via the Born rule to satisfy (1.2). The formalism then decides whether the quantum-to-classical transition happens: does the state $|\psi\rangle$ becomes localized in phase space? The goal of this paper is to show that the conditioned state of the perturbations does become classical driven by interaction with the environment and the inflationary expansion.

It is well known that there are an infinite number of ways to write the solution of a master equation ρ as a stochastic average as in (1.2), each known as an *unravelling*.[4] However, there is a particular unravelling that follows from implementing the Born rule to a subsystem, the perturbations in the present context. This is the *Born unravelling* defined in [17] and first described by Diósi [18,19].[5] This has a phenomenology that is similar to another unravelling, known as *quantum state diffusion* [27–29] which has been widely studied as a description of the quantum-to-classical transition in [30–38]. In both unravellings, the quantum-to-classical transition happens dynamically when the conditioned state becomes sufficiently localized that Ehrenfest's theorem applies and an effective description in terms of a position in phase space applies. In [17] it was argued that the quantum-to-classical transition becomes a dynamical process that involves the following conceptual steps:



- The unconditioned state ρ of the subsystem of interest satisfies a master equation, within the Born–Markov approximation.
- The conditioned state $|\psi\rangle$ (the state from the frame of reference of the subsystem) satisfies a particular unravelling of this master equation which takes the form of deterministic evolution with a non-linear, non-Hermitian, Hamiltonian, interspersed with stochastic jumps into orthogonal states (arising from applying the Born rule to each coherent interaction of the system with the environment).
- Under favourable conditions, the dynamics of the conditioned state drive it to become localized on macroscopic scales and Ehrenfest's Theorem can be invoked.
- The localized state can be described by point in phase space (i.e. a classical state) evolving according to the classical equations of motion plus stochastic noise, i.e. a Langevin equation.
- Finally, to bring things full circle, the Langevin equation has an associated Fokker–Planck equation whose solution is identified with the Wigner function of the unconditioned state in the semi-classical limit.

The purpose of this work is to apply this formalism to the cosmological perturbations by considering their evolution according to the Born unravelling. We will argue that, with a suitable coupling to the environment, although the unconditioned state spreads out in field space when a mode exits the horizon during inflation, the conditioned state is driven to become increasingly localized in field space as a result of the expansion (just as described above). In the end the usual state analysed in the literature—the unconditioned state—becomes a probability density for the conditioned state that is effectively specified by a point in field space. We can follow the stochastic evolution of this state and find that it follows a random walk in field space once the mode under discussion has crossed the horizon. The CMB across the sky can be viewed as an ensemble of endpoints of the classical stochastic process.

The scalar curvature perturbations ζ are effectively described by a scalar field $\mathrm{v} = \sqrt{2\varepsilon}a\zeta$, the Mukhanov–Sasaki variable, each Fourier mode of which is effectively a parametric oscillator whose Schrödinger equation looks like that of non-relativistic quantum mechanics[6]:

$$-\frac{\partial^2\psi}{\partial \mathrm{v}^2} + \omega^2 \mathrm{v}^2\psi = 2i\frac{\partial\psi}{\partial\tau}\,. \tag{1.3}$$

Here, v is identified with either of the real combinations $(\mathrm{v}_{\boldsymbol{k}} + \mathrm{v}_{-\boldsymbol{k}})/\sqrt{2}$ or $i(\mathrm{v}_{\boldsymbol{k}} - \mathrm{v}_{-\boldsymbol{k}})/\sqrt{2}$, of wave vector $\boldsymbol{k}$, and τ is the conformal time during inflation. The latter has negative values and approaches τ_{end} at the end of inflation. A mode exits the horizon when $k|\tau| \sim 1$ and the modes of interest for the CMB and structure formation underwent $\Delta N \sim 50$ e-folds before the end of inflation, so $k|\tau_{\text{end}}| \sim e^{-\Delta N}$ for the modes of interest. Above, $a(\tau)$ is the scale factor.

The power spectrum of the scalar curvature perturbations is simply related to the variance of the quantum mechanical problem,[7]

$$\Delta_\zeta^2 = \frac{k^3}{2\pi^2}\langle\zeta\zeta\rangle = \frac{k^3}{4\pi^2\varepsilon a^2}\langle \mathrm{v}^2\rangle\,. \tag{1.4}$$

In the above,

$$\omega(\tau)^2 = k^2 - \frac{(a\sqrt{2\varepsilon})''}{a\sqrt{2\varepsilon}}\,, \tag{1.5}$$

where ε is a slow roll parameter. For present purposes, we will ignore slow roll effects and assume an exact de Sitter geometry during inflation for which $a = -1/(H\tau)$, for constant H, so $\omega^2 = k^2 - 2/\tau^2$ and $\Delta_\zeta^2 = k^3H^2\tau^2\langle \mathrm{v}^2\rangle/4\pi^2\varepsilon$.

The initial conditions of the mode are that at early times, $k|\tau| \gg 1$, the mode sits in the ground state of the oscillator with $\omega = k$, the Bunch–Davies vacuum,

[4] The terminology comes from the theory of *quantum trajectories* that describes the behaviour of a subsystem conditioned on the measurements made on it [13–16].

[5] See also [20–23]. As shown in [20], the Born unravelling also defines a set of *consistent histories* in the formalism of [24–26].

[6] We present the Schrödinger equation in a form that looks like a harmonic oscillator. The Hamiltonian is related to the Hamiltonian of the perturbations by a canonical transformation that just shifts the momentum $\pi \to \pi - (a'/a)\mathrm{v}$. So before shifting, we have (classically) $\pi = \mathrm{v}'$ while after shifting $\pi = \mathrm{v}' - (a'/a)\mathrm{v} = (\sqrt{2\varepsilon}a)\zeta'$.

[7] In these formulae, we are ignoring the delta functions of the wave vector.