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Breaking flavor democracy with symmetric perturbations

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ABSTRACT

Flavor democracy broken in the fermion mass matrix by means of small perturbations can give rise to hierarchical fermion masses. We study the breaking of the $\mathbb{S}_3^L \times \mathbb{S}_3^R$ symmetry associated with democratic mass matrices to a smaller exchange symmetry $\mathbb{S}_2^L \times \mathbb{S}_2^R$ in the charged lepton, up and down quark sectors. An additional breaking of the $\mathbb{S}_2^L \times \mathbb{S}_2^R$ symmetry is necessary for the down quark mass matrix, which yields arbitrary perturbations in that sector. On the other hand, we require the neutrino mass matrix to be diagonal at the leading order, with the perturbations left arbitrary due to the absence of any guiding symmetry. We show that the interplay between these two kinds of perturbations reproduces the quark and lepton mass and mixing observables for either hierarchy of neutrino masses.

1. Introduction

Flavor mixing has long been established in both quark and lepton sectors. However, the standard model (SM) of particle physics does not offer an explanation of the observed flavor structure in either sector. In fact, the Yukawa couplings of the recently discovered Higgs boson are the least understood part of the SM. Over the years various attempts have been made to explain the flavor structure of the SM fermions. In principle, one can follow either a top-down or a bottom-up [1–3] approach. In the former case, one starts by imposing a flavor symmetry to the theory, followed by prescriptions to break that symmetry to generate the Cabibbo–Kobayashi–Maskawa (CKM) and Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrices. The observed mixing patterns are then explained in terms of some residual symmetry. In contrast, in the latter case one attempts to reconstruct residual symmetries from the mixing patterns in both up-type and down-type fermion sectors [4], and then the full flavor symmetry is obtained as a product group of residual symmetries [5]. This is called the *phenomenological mass matrix approach*. Residual symmetries, however, control the mixing pattern in both cases [6] and can establish a sum rule [7] between the Dirac CP phase and mixing angles. We adopt the second approach in this paper.

It is well known that the mixing pattern in the lepton and quark sector differ significantly. While the lepton sector exhibits large mixing angles, the mixing angles in the quark sector are small. The observed lepton and quark mixings are a combination of mixings in the down and up-type fermions, with $V_{\rm PMNS}$ and $V_{\rm CKM}$ in the lepton and quark sectors given by $V_\ell^\dagger V_\nu$ and $V_u^\dagger V_d$, respectively. Although the disparity between the above mixing patterns may appear puzzling, a unified framework can be constructed if both up- and down-type quark matrices and one of the lepton or neutrino mass matrices possess the same mixing pattern. Then large mixing angles can be generated in $V_{\rm PMNS}$, and at the same time keeping $V_{\rm CKM}$ close to the identity.

Approximate democratic mass matrices [8–16] for both up- and down-type quarks offer an exciting avenue to explain the small CKM mixing angles and the large hierarchy of quark masses. The application of the same hypothesis to both charged lepton and neutrino mass matrices leads to too small mixing angles in $V_{\rm PMNS}$. Instead, if the neutrino mass matrix assumes an almost diagonal form, then $V_{\rm PMNS}$ can easily have two large mixing angles [2,3]. The charged lepton mass matrix remains democratic, leading to a natural explanation of the hierarchy in their masses, with $m_1 = m_2 = 0$ at the leading order. However, one needs to break the residual $\mathbb{S}_3$ symmetries in the up, down, and charged lepton mass matrices to accommodate nonzero light fermion masses and to fit the experimentally measured mixing angles.

Such a study is performed in Ref. [17] under the assumption that there is no remnant symmetry in the up, down, and charged lepton mass matrices, after the $\mathbb{S}_3^L \times \mathbb{S}_3^R$ symmetry in the democratic matrices is broken. Hence, there is no guiding symmetry to regulate the small perturbations, and the perturbations are random. In contrast, we consider a residual $\mathbb{S}_2^L \times \mathbb{S}_2^R$ symmetry in the

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up, down and charged lepton mass matrices [11]. However, the $\mathbb{S}_2^L \times \mathbb{S}_2^R$ symmetry in the down sector needs to be broken eventually. Also, we assume that there is no residual symmetry left in the neutrino sector allowing the perturbations in that sector to be arbitrary. We take neutrinos to be Majorana particles. (Previously, Ref. [16] studied $\mathbb{S}_3^L \times \mathbb{S}_3^R \to \mathbb{S}_2^L \times \mathbb{S}_2^R$ breaking with different subsequent breaking patterns than we consider.) Our approach is different from the anarchy scenario [18] in which the mass matrix elements are unconstrained.

The paper is organized as follows. In Section 2 we review the origin and predictions of democratic mass matrices. We discuss the consequences of breaking the $\mathbb{S}_3^L \times \mathbb{S}_3^R$ to a smaller $\mathbb{S}_2^L \times \mathbb{S}_2^R$ symmetry in Section 3, and show that solutions can be found that are consistent with all mass and mixing constraints in the lepton and quark sectors. We conclude in Section 4.

2. Democratic mass matrix

Without Yukawa matrices the SM has an $U(3)^5$ global symmetry. One can start building a model with flavor structure by using a maximal subgroup of $U(3)^5$. Following Ref. [3], our starting point is $O(3)_{L\{Q,L\}} \times O(3)_{R\{u^c,d^c,e^c\}}$. In the above model, the $O(3)_L$ symmetry in the neutrino sector is explicitly broken by Σ_L, which transforms as $(\mathbf{5},\mathbf{1})$ under the symmetry groups and takes the form diag$\{a, b, -a-b\}$. This gives a diagonal neutrino mass matrix at the leading order. Besides, the explicit breaking of the flavor symmetry prevents the existence of unwanted Nambu–Goldstone bosons. On the other hand, the charged leptons obtain mass at the leading order from another explicit breaking of $O(3)_L \times O(3)_R$ by $\phi_{L,R}$, which transform as $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$, respectively. The vacuum expectation values (VEV) of $\phi_{L,R}$ have a structure $\langle\phi_{L,R}\rangle = (1,1,1)^T v_{L,R}$, resulting in a democratic mass matrix for the charged leptons preserving an $\mathbb{S}_3^L \times \mathbb{S}_3^R$ symmetry. The up and down-type quarks obtain mass at the leading order similarly to charged leptons.

The democratic fermion mass matrix, possessing $\mathbb{S}_3^L \times \mathbb{S}_3^R$ symmetry has the form [8–16],

$$M_f = \frac{M_0}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{2.1}$$

where M_0 is the characteristic mass scale [2,3]. This matrix can be diagonalized by $M_f = V_L D_f V_R^\dagger$, where V_L (V_R) are mixing matrices for left (right) handed fermions, and $D_f = \mathrm{diag}\{m_1, m_2, m_3\}$ is the diagonalized mass matrix. Now, V_L can be determined by diagonalizing $M_f M_f^\dagger = V_L D_f^2 V_L^\dagger$, where $M_f M_f^\dagger$ has the same form as M_f with a normalization constant $M_0^2/3$. Thus, by diagonalizing $M_f M_f^\dagger$ the most general form of V_L is

$$V_L^\dagger = \begin{pmatrix} e^{i\beta_1} & & \\ & e^{i\beta_2} & \\ & & e^{i\beta_3} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \equiv R V_0, \tag{2.2}$$

where each mass eigenvalue is associated with a rephasing degree of freedom denoted by β_i ($i = 1,2,3$). It is obvious from Eqs. (2.1) and (2.2) that the squared eigenvalues generated by diagonalizing $M_f M_f^\dagger$ are $\{0, 0, M_0^2\}$, thus producing a natural hierarchy in the respective fermion sectors.

In the lepton sector, since $V_\nu = I$ at the leading order, Eq. (2.2) is the PMNS matrix. The mixing angles in terms of the standard PMNS parametrization are [12,15]

$$\theta_{13} = 0^\circ, \quad \theta_{12} = 45^\circ, \quad \theta_{23} = 54.7^\circ, \tag{2.3}$$

which are clearly in tension with the measured values of the mixing angles for neutrinos [20]. Hence, it is imperative to break the democracy in the charged lepton mass matrix of Eq. (2.1). This is also needed to provide a nonzero mass to m_μ.

On the other hand, in the quark sector both up- and down-type quark mass matrices have a democratic form, so that $V_{\rm CKM}$ is simply the identity matrix with no rephasing matrices. Also, $m_{1,2} = 0$ in both quark sectors. Again, we need a deviation from democracy in the quark sector to explain the observed quark masses and mixing angles.

3. Broken democracy

In this section, we discuss the consequences of breaking the democracy in the mass matrices with $\mathbb{S}_2^L \times \mathbb{S}_2^R$ invariant perturbations, and comment on perturbations preserving other smaller discrete exchange symmetries.

It is reasonable to assume that once the $\mathbb{S}_3^L \times \mathbb{S}_3^R$ symmetry in the democratic mass matrix is broken, there will not be any remnant symmetry in the charged lepton mass matrix. In Ref. [17], anarchic random perturbations are employed to write $M_{f_{ij}} = \frac{M_0}{3}(1+\delta_{ij})$, where $0 \le |\delta_{ij}| \le \delta_{max}$ and the phases of δ_{ij} are randomly scanned. The peak of the mixing angle and Dirac CP phase distributions are found to be stable against different values of δ_{max}.

In contrast, we assume that the $\mathbb{S}_3^L \times \mathbb{S}_3^R$ symmetry in the democratic mass matrix breaks to a smaller exchange symmetry $\mathbb{S}_2^L \times \mathbb{S}_2^R$. Thus, the perturbations to the charged lepton and quark mass matrices are $\mathbb{S}_2^L \times \mathbb{S}_2^R$ invariant in our model. Although $m_e = 0$ in this case, the measured electron mass can be easily generated by radiative corrections. The general form of charged lepton, up, and down quark mass matrices with $\mathbb{S}_2^L \times \mathbb{S}_2^R$ perturbations is given by

$$M_f = \frac{M_0}{3}\left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} a & a & b \\ a & a & b \\ c & c & d \end{pmatrix}\right], \tag{3.1}$$

where $\{a, b, c, d\} \in \mathbb{C}$. As usual, to obtain the PMNS matrix we need to diagonalize $M_f M_f^\dagger$. We do this sequentially to get some insight about the mass matrix. First, we reduce $M_f M_f^\dagger$ to the form,

$$\tilde{M}_f^2 = \frac{M_0^2}{9}\begin{pmatrix} 0 & 0 & 0 \\ 0 & X & Y \\ 0 & Y^* & 9+Z \end{pmatrix}, \tag{3.2}$$

by evaluating $V_0 M_f M_f^\dagger V_0^\dagger$, where V_0 is defined in Eq. (2.2). Here, X, Y and Z are

$$\begin{aligned} X &\equiv \frac{2}{3}\left[|b|^2 + |d|^2 + 2|a-c|^2 - 2Re(bd^*)\right], \\ Y &\equiv \frac{\sqrt{2}}{3}\left[|d|^2 + 2|c|^2 - 4|a|^2 - 2|b|^2 - 3(2a+b-2c-d)\right. \\ &\quad \left. + 4a^*c - 2ac^* + 2b^*d - bd^*\right], \\ Z &\equiv \frac{2}{3}|2a+c|^2 + \frac{1}{3}|2b+d|^2 + 2Re(4a+2b+2c+d). \end{aligned} \tag{3.3}$$

From Eq. (3.2) it is obvious that $m_e = 0$ with the remnant $\mathbb{S}_2^L \times \mathbb{S}_2^R$ symmetry. Now, $\tilde{M}_f^2$ can be diagonalized by $\tilde{M}_f^2 = T^\dagger D_f^2 T$, where