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Short range correlations and the isospin dependence of nuclear correlation functions

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ABSTRACT

Pair densities and associated correlation functions provide a critical tool for introducing many-body correlations into a wide-range of effective theories. *Ab initio* calculations show that two-nucleon pair-densities exhibit strong spin and isospin dependence. However, such calculations are not available for all nuclei of current interest. We therefore provide a simple model, which involves combining the short and long separation distance behavior using a single blending function, to accurately describe the two-nucleon correlations inherent in existing *ab initio* calculations. We show that the salient features of the correlation function arise from the features of the two-body short-range nuclear interaction, and that the suppression of the *pp* and *nn* pair-densities caused by the Pauli principle is important. Our procedure for obtaining pair-density functions and correlation functions can be applied to heavy nuclei which lack *ab initio* calculations.

1. Introduction

Correlation functions are a valuable tool for describing interacting many-body systems, providing a means of encapsulating complex many-body dynamics. In the absence of correlations, a many-body probability density, such as that from a many-body quantum mechanical wave-function, can be written as an anti-symmetrized product of single-particle probability densities. The correlation function describes important deviations from this picture. Our aim here is to explain the basic physics inputs that determine the nuclear pair-density functions and the correlation functions derived from them. This is done by blending the short-distance behavior, as determined by the contact formalism [1–3], with the known long distance behavior. The input needed to use the contact formalism is accessible from experimental data, as shown in Ref. [2].

Correlation functions are widely used in nuclear physics. For recent reviews see Refs. [4,5]. The nucleus is a strongly-interacting, quantum mechanical, many-body system with high density and a complicated interaction between constituent nucleons. There is no fundamental central potential, so correlations must exist. An early paper that modeled nuclear correlation functions [6] was used in a wide variety calculations (see the early review [7]) involving the strong and weak interactions, demonstrating the impact of correlation functions on the field. More recent examples in which correlation functions are crucial ingredients include: calculations of neutrinoless double beta decay [8–13], nuclear transparency in quasielastic scattering [14–19], shadowing in deep inelastic scattering [20], and parity violation in nuclei [21,22].

Despite the wide use of correlation functions, their spin and isospin dependence has received less attention. The nucleon–nucleon interaction is both spin and isospin dependent, and these dependencies become very important at short-range, leading to phenomena such as the strong preference for proton–neutron short-range correlated pairs [23–29].

The calculations in this paper use the formalism of nuclear contacts [2,3] to determine the spin and isospin decomposition of the two-body density that determines the correlation function. This formalism is based on the separation of scales inherent in the long- and short-range structure of nuclei [2,3]. At short distances, the aggregate effect of long-range interactions can be encapsu-

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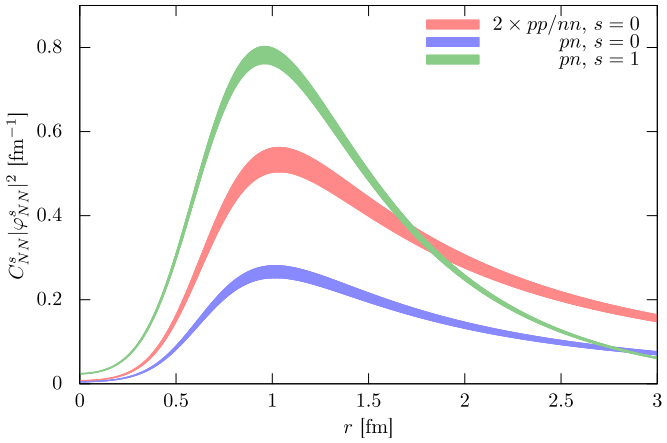

Fig. 1. In the two-body density from contact formalism [2,3], the *np* two-body density is dominated by spin-1 pairs. ^{40}Ca, shown here, illustrates this universal behavior. For $r \leq 0.9$ fm, these results reproduce those of Cluster Variational Monte Carlo (CVMC) [34] calculations. The *pp*/*nn* spin-0 density (peak value 0.5) is enhanced by a factor of 2 to provide some separation from *np* spin-0. The pn density peaks at 0.2 for spin 0, and 0.8 for spin 1 . (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

lated into coefficients, called "contacts," which are nucleus-specific, while the underlying short-range behavior is a universal property of the two-body nuclear interaction. In the contact formalism, the two-body density, $\rho_{NN,s}(r)$, defining the probability for finding a nucleon–nucleon pair with separation distance r, can be modeled at short distance ($r \lesssim 1$ fm) by:

$$\rho^{\text{contact}}_{NN,s}(r) = C_A^{NN,s} \times |\varphi_{NN,s}(r)|^2 \tag{1}$$

for nucleus, A, where C_A is the contact coefficient, NN stands for proton–proton (pp), proton–neutron (pn), or neutron–neutron (nn) pairs and the index s denotes the spin $0, 1$ of the two-nucleon systems. The wave functions $\varphi_{NN,s}(r)$ are zero-energy (S- or S-D wave) solutions to the Schrödinger equation with a modern nucleon–nucleon potential, e.g., AV18 [30]. Equation (1) assumes angle averaging, and the zero-energy nature restricts the number of contacts. The key assumption in this formalism is that these functions, $\varphi_{NN,s}(r)$ can be used for all nuclei. Contact coefficients can be determined for the different possible spin and isospin configurations of a nucleon–nucleon pair from experiment or from fitting *ab initio* calculations. Previous studies [2], show that the NN state with deuteron quantum numbers is dominant: the peak value of the product $C_A^{np,s=1}|\varphi_{np,s=1}(r)|^2$ is four times larger than for any other combination. This dominance is caused by the tensor force [31–33] As an example, the decomposition of the two-body density from contact formalism for ^{40}Ca is shown in Fig. 1.

2. Describing the pair (two-body) density

The two-body pair density distribution $\rho_{NN,s}(\vec{r})$, is defined as the probability density for finding a nucleon–nucleon pair separated by $\vec{r}$, with relative spin s, normalized so that its integral is the number of possible NN, s pairs. The two-body density is expressed as a matrix element of the nuclear wave function $|\psi\rangle$ by

$$\rho_{NN,s}(\vec{r}) \equiv \sum_{\substack{i,j \in NN \\ i<j}} \langle\psi|\delta(\vec{r} - \vec{r}_{ij})P_s|\psi\rangle, \tag{2}$$

where $\vec{r}_{ij}$ is the separation between nucleons i and j and P_s is a projection operator onto the spin s of the nucleon pair.

Our aim here is to provide a simple understanding of the underlying mechanisms that produce the isospin dependence and other features. We will compare our results for $\rho_{NN}(r)$ to *ab initio* calculations performed using Cluster Variational Monte Carlo (CVMC) [34] of ^{16}O and ^{40}Ca, the two heaviest nuclei studied so far using CVMC [35]. Several other calculations that include the necessary spin and isospin dependence in computing densities are those of Refs. [31,34,36–39]. A nice *ab initio* treatment of light nuclei has recently appeared [40]. See also Ref. [41], which is based on nuclear matter calculations.

To achieve the desired understanding we design a model in which the two-body density is formed from a combination of the correlated density coming from nuclear contact formalism (Fig. 1), which accounts for the behavior for $r \leq 0.9$ fm and a longer-ranged term, $\rho^{(0)}_{NN}(r)$, for which correlations are expected to be unimportant. We define this term as $\rho^{(0)}_{NN}(r)$, given by

$$\rho^{(0)}_{NN}(\vec{r}) \equiv S_{NN} \int \mathrm{d}^3\vec{R}\,\rho_N(\vec{R} + \vec{r}/2)\rho_N(\vec{R} - \vec{r}/2), \tag{3}$$

where ρ_N is the one-body density, normalized to proton or neutron number, $\vec{R}$ represents the center-of-mass position of a nucleon–nucleon pair, and S_{NN} represents a symmetry factor, which equals 1 for pn pairs, equals $Z(Z-1)/2Z^2$ for pp pairs – since there are only $Z(Z-1)/2$ unique pp pairs in a nucleus – and equals $N(N-1)/2N^2$ for nn pairs.

Then the full two-body density combines the short and long distance behavior, with the relative weighting determined by a blending function, $g_{NN}(r)$, and constant, κ, such that

$$\rho_{NN}(r) = g_{NN}(r)\rho^{\text{contact}}_{NN}(r) + \kappa(1 - g_{NN}(r))\rho^{(0)}_{NN}(r). \tag{4}$$

We can understand how the correlated and uncorrelated densities contribute to produce the specific behavior of the correlation function seen through CVMC by assessing the quality of this model and by determining the blending function.

In order to parameterize $g_{NN}(r)$, we consider the short- and long-range constraints. At short-distance, where $\rho^{\text{contact}}_{NN}(r)$ is an accurate description of the two-body density [2], $g_{NN}(r)$ equals 1. For large distances, ρ_{NN} must approach $\rho^{\text{uncorr.}}_{NN}$. Since $\rho^{\text{contact}}_{NN}$ falls off approximately as $1/r^2$ for $r > 2$ fm, g_{NN} must approach $(\kappa - 1)/\kappa$ in the long-range limit, in order that the pair density approach $\rho^{(0)}_{NN}$. We propose the following model which meets these requirements:

$$g_{NN}(r) = \begin{cases} 1 & r \leq 0.9 \text{ fm}, \\ \frac{1}{\kappa}\left(\kappa - 1 + e^{(0.9\text{ fm} - r)/a}\right) & r > 0.9 \text{ fm}. \end{cases} \tag{5}$$

For $r < 0.9$ fm, $\rho_{NN}(r)$ is modeled well by the contact expression Eq. (1) (see [2]). For $r > 0.9$ fm, the contact density and the uncorrelated densities are blended, with a characteristic length-scale, a. In principle, a would depend on the isospin of the pairs and on the specific nucleus being studied.

Varying the parameters of Eq. (5) to describe pp, nn and pn pairs in ^{16}O and ^{40}Ca shows that the same blending function $g(r)$ can be used to describe all the two-body densities calculated using CVMC, shown in Fig. 2. CVMC correlation functions are shown as points, while our model, described in equation (4), is shown with bands, for which the dominant contribution to the uncertainty comes from the contact coefficients, C_{NN}. The uncorrelated density, $\rho^{(0)}_{NN}$, used by our model is supplied by CVMC calculations of the one-body density ρ_N. The residuals show the difference between the CVMC density and those of the model, divided by the model, with the error bars showing the uncertainties in the CVMC densities. Our model is able to reproduce the correlation functions for both pp and pn pairs in two different nuclei (as these CVMC calculations treat p and n symmetrically, and since ^{16}O and ^{40}Ca