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Mu-tau symmetry and the Littlest Seesaw

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ABSTRACT

Motivated by the latest neutrino oscillation data which is consistent with maximal atmospheric mixing and maximal leptonic CP violation, we review various results in $\mu\tau$ symmetry, then include several new observations and clarifications, including identifying a new general form of neutrino mass matrix with $\mu\tau$ symmetry. We then apply the new results to the neutrino mass matrix associated with the Littlest Seesaw model, and show that it approximately satisfies the new general form with $\mu\tau$ symmetry, and that this is responsible for its approximate predictions of maximal atmospheric mixing and maximal CP violation in the lepton sector.

1. Introduction

Although neutrino oscillation experiments have provided the first laboratory evidence for new physics beyond the Standard Model (BSM) in the form of neutrino mass and mixing [1], the nature of neutrino mass and lepton flavour mixing remains unknown [2,3]. While T2K consistently prefers an almost maximal atmospheric mixing angle [4], NOνA originally excluded maximal mixing at 2.6σ CL [5], but the latest analysis with more data is now consistent with maximal mixing [6–9]. Furthermore, although the CP violating Dirac phase relevant for neutrino oscillations has not been directly measured, the global analyses seem to favour somewhat close to maximal values for this phase as well. The latest neutrino data therefore seems to be consistent with the hypothesis of maximal atmospheric mixing and maximal CP violation in the lepton sector. This could either be a coincidence, or may be pointing towards some underlying symmetry or structure underpinning the lepton flavour sector.

The leading candidate for a theoretical explanation of neutrino mass and mixing remains the seesaw mechanism [10–14]. However the seesaw mechanism involves a large number of free parameters at high energy, and is therefore difficult to test. One approach to reducing the number of seesaw parameters is to consider the minimal version involving only two right-handed neutrinos (2RHN), first proposed by one of us [15,16]. In such a scheme the lightest neutrino is massless. An early simplification [17], involved two texture zeros in the Dirac neutrino mass matrix consistent with cosmological leptogenesis [18–25]. Although the normal hierarchy (NH) of neutrino masses, favoured by current data, is incompatible with the 2RHN model with two texture zeros [24,25], the one texture zero case originally proposed [15,16] remains viable.

Recently the Littlest Seesaw (LSS) model has been proposed as a particular type of 2RHN model with one texture zero, which also postulates a well defined Yukawa structure with a particular constrained structure involving just two independent Yukawa couplings [26–31], leading to a highly predictive scheme. Interestingly, the LSS model predicts close to maximal atmospheric mixing and CP violation, as favoured by current data, a result which can be understood from analytic results.

On the other hand, traditionally, the predictions of maximal atmospheric mixing arise from the notion of interchange symmetry between the muon neutrino ν_μ and the tau neutrino ν_τ, namely $\nu_\mu \leftrightarrow \nu_\tau$, known as $\mu\tau$ interchange symmetry in the neutrino sector. When combined with CP symmetry, such a $\mu\tau$ symmetry, also known as $\mu\tau$ reflection symmetry, can also lead to maximal CP violation. For a review of $\mu\tau$ symmetry see e.g. [32] and references therein.

In this paper, motivated by the latest neutrino oscillation data which is consistent with maximal atmospheric mixing and CP violation, we give a timely survey of various results in $\mu\tau$ symmetry, making several new observations and clarifications along the way. We then apply the new results to the neutrino mass matrix associated with the Littlest Seesaw model, and show that it approximately satisfies a general form of $\mu\tau$ symmetry, and that this

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is responsible for its approximate predictions of maximal atmospheric mixing and maximal CP violation in the lepton sector.

The layout of the remainder of this paper is as follows. In section 2 we introduce and define different types of $\mu\tau$ symmetry as applied to the PMNS matrix V, the neutrino mass matrix M_ν, and its hermitean square $H_\nu \equiv M_\nu^\dagger M_\nu$. In section 3 we give basis invariant conditions on H_ν leading to maximal atmospheric mixing and maximal CP violation. In section 4 we present a general form for M_ν with $\mu\tau$ symmetry leading to maximal atmospheric mixing and maximal CP violation. In section 5 we show how the $\mu\tau$ conjugation operation can be useful for relating different neutrino mass matrices which have the general form of $\mu\tau$ symmetry. In section 6 we apply the results to the LSS mass matrix and show why this model has approximate $\mu\tau$ symmetry. In section 7 we discuss accidental implementations of $\mu\tau$ symmetry and give an example. Finally section 8 concludes the paper. The Appendices contain some of the proofs of results in the paper. Appendix A provides a proof that a $\mu\tau$ symmetric H_ν implies and is implied by $\mu\tau$ symmetric PMNS mixing. Appendix B makes the connection of the general form of M_ν with $\mu\tau$ symmetry with CP transformations.

2. Other types of $\mu\tau$ symmetry: $\mu\tau$-U and $\mu\tau$-R

Let us denote by $\mu\tau$ universal ($\mu\tau$-U) mixing the PMNS matrix V characterized by the following two conditions: (i) fully nonvanishing first row,

$$|V_{ej}| \neq 0\,, \quad j = 1, 2, 3, \tag{1}$$

and (ii) equal moduli for the μ (second) and τ (third) rows [33,34],

$$|V_{\mu j}| = |V_{\tau j}|\,, \quad j = 1, 2, 3. \tag{2}$$

In other words the modulus of the $\mu\tau$-U PMNS matrix elements have the form

$$|V| = \begin{pmatrix} |V_{e1}| & |V_{e2}| & |V_{e3}| \\ |V_{\mu 1}| & |V_{\mu 2}| & |V_{\mu 3}| \\ |V_{\mu 1}| & |V_{\mu 2}| & |V_{\mu 3}| \end{pmatrix}. \tag{3}$$

One can show within the standard parametrization that conditions (1) and (2) are equivalent to having nonzero θ_{13} together with maximal atmospheric angle and Dirac CP phase[1]:

$$\theta_{13} \neq 0\,, \quad \theta_{23} = 45^\circ\,, \quad \delta_{\rm CP} = \pm 90^\circ\,, \tag{4}$$

which are consistent with current data. The condition (1) ensures the first inequality while (2) ensures the rest. In fact, condition (1) implies that both θ_{13} and θ_{12} are nontrivial (different from 0 or $\pi/2$). Notice that the Majorana phases in V are not constrained.

Harrison and Scott [33] showed that, allowing rephasing transformations from the left and from the right,[2] any $\mu\tau$-U PMNS mixing matrix V can be cast in the form

$$V_0 = \begin{pmatrix} |V_{e1}| & |V_{e2}| & |V_{e3}| \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\mu 1}^* & V_{\mu 2}^* & V_{\mu 3}^* \end{pmatrix}. \tag{5}$$

Moreover, when all $|V_{ej}|$ are nonzero, i.e., condition (1) is valid, it is guaranteed that not all of the phases in $V_{\mu i}$ can be removed and V_0 is essentially complex. This fact is consistent with the presence of CP violation in (4). The form (5) can be easily checked by imposing maximal angle and phase in (4) in the standard parametrization and applying appropriate rephasing transformations; see Ref. [36] for the explicit form. In Ref. [33] a different proof was originally supplied and the restriction (1) was not imposed.

Instead of characterizing the mixing matrix, it is often more interesting to characterize the neutrino mass matrix M_ν that is responsible for the mixing in the flavor basis where the $\mu\tau$-U PMNS matrix comes from the diagonalization of the neutrino mass matrix. As condition (2) is insensitive to Majorana phases, it is useful to consider the hermitean square $H_\nu \equiv M_\nu^\dagger M_\nu$ of the neutrino mass matrix M_ν for both Majorana or Dirac neutrinos.

We say a hermitean or symmetric 3×3 matrix A is $\mu\tau$-reflection ($\mu\tau$-R) symmetric[3] if

$$P_{\mu\tau} A P_{\mu\tau} = A^*\,, \tag{6}$$

where

$$P_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{7}$$

represents $\mu\tau$ interchange. According to this definition, the hermitean square mass matrix $H_\nu = H_\nu^\dagger$ is $\mu\tau$-R symmetric [33] if it has the form

$$H_\nu = \begin{pmatrix} A & D & D^* \\ D^* & B & C^* \\ D & C & B \end{pmatrix}, \tag{8}$$

with A, B real and positive while C, D should have irremovable phases ($\text{Im}[C^* D^2] \neq 0$). It can readily be checked that, if the hermitean square mass matrix H_ν is $\mu\tau$-R symmetric in the flavour basis (i.e. has the form in Eq. (8)), then this leads to a $\mu\tau$-U PMNS matrix, with the usual predictions of maximal atmospheric mixing and maximal CP violation. In fact it can be proved that a $\mu\tau$-U PMNS matrix implies and is implied by H_ν being $\mu\tau$-R symmetric in the flavour basis (see Appendix A).

For Majorana neutrinos, the complex symmetric mass matrix M_ν which leads to a $\mu\tau$-R symmetric hermitean square mass matrix H_ν (and hence $\mu\tau$-U PMNS matrix) may take the following special $\mu\tau$-R symmetric form [34][4]

$$M_\nu = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix}, \tag{9}$$

with real a, b and $\text{Im}[c^* d^2] \neq 0$. It can readily be checked that the mass matrix of the special $\mu\tau$-R symmetric form in (9) leads to a $\mu\tau$-R symmetric hermitean square mass matrix H_ν as in (8) when the hermitean square is taken (and hence a $\mu\tau$-U PMNS matrix). However it is not necessary for M_ν to be $\mu\tau$-R symmetric, in order to lead to a $\mu\tau$-R symmetric hermitean square mass matrix H_ν.[5] Unlike Ref. [33], we shortly show that, while $\mu\tau$-U PMNS mixing is equivalent to having a $\mu\tau$-R symmetric H_ν, it is not equivalent to having a $\mu\tau$-R symmetric M_ν in the case of Majorana neutrinos. In other words, (9) is not the most general form of neutrino mass matrix with $\mu\tau$ symmetry. But, before giving that, we first discuss the basis invariant conditions on H_ν with $\mu\tau$ symmetry.

[1] Also denoted as cobimaximal mixing in Ref. [35].

[2] The following rephasing freedom from the left still survives: $V_{\mu k} \to e^{i\alpha} V_{\mu k}$, $V_{\tau k} \to e^{-i\alpha} V_{\tau k}$.

[3] Also denoted as $\text{CP}^{\mu\tau}$ in Ref. [40].

[4] This form resulting from a model was first proposed in Ref. [37].

[5] These points were already made in Refs. [38,39] but here we extend their analysis.