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Tomography by neutrino pair beam

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ABSTRACT

We consider tomography of the Earth's interior using the neutrino pair beam which has recently been proposed. The beam produces a large amount of neutrino and antineutrino pairs from the circulating partially stripped ions and provides the possibility to measure precisely the energy spectrum of neutrino oscillation probability together with a sufficiently large detector. It is shown that the pair beam gives a better sensitivity to probe the Earth's crust compared with the neutrino sources at present. In addition we present a method to reconstruct a matter density profile by means of the analytic formula of the oscillation probability in which the matter effect is included perturbatively to the second order.

1. Introduction

Our understanding of neutrino has improved greatly since the end of the last century. The observation of flavor oscillations of neutrinos has shown the presence of non-zero masses of neutrinos contrary to the prediction of the Standard Model. This is a clear signature of new physics beyond the Standard Model. Thanks to the remarkable efforts of various neutrino experiments, the mass squared differences and mixing angles of (active) neutrinos have been measured very precisely at present [1]. The origin of neutrino masses is, however, unknown and then it is important to investigate the fundamental theory of neutrino physics. Moreover, it should be useful to consider seriously the application of neutrino physics to various fields of basic science.

In the Standard Model neutrinos are unique matter particles which possess only the weak interaction (in addition to gravity), and their interaction rates are very suppressed accordingly. It is found that most of them can penetrate the Earth without a scattering when the energy is smaller than $\mathscr{O}(10^5)$ GeV [2]. This shows that neutrinos can be used to probe the deep interior of the Earth.

The idea of the neutrino tomography has been pointed out in the 1970s [3,4]. By measuring the absorption rates of neutrinos passing through the object from different angles, the image of the object can be reconstructed. This is similar to the computed tomography using x-rays, which enables us to probe inside solids without destruction. This method is called as the neutrino absorption tomography [3–24]. In addition to this there have been proposed two other methods of neutrino tomography. One is the method using neutrino oscillations [25–45], and the other is the tomography using neutrino diffraction [46,47].

The former one utilizes the energy spectrum of the neutrino oscillation probability, which is distorted, compared to the vacuum one, by the interaction with matter through which neutrinos pass from the production to the detection point. The distortion pattern depends on the profile of the number density of electron in matter, that can be translated into the matter density profile by assuming the charge neutrality and the equality of neutron and proton numbers in matter. It is then possible to probe the deep interior of the Earth by measuring the oscillation probability at the sufficient accuracy.

In this letter we revisit the possibility to realize the neutrino oscillation tomography. The main difficulties of its feasibility include the lack of the powerful neutrino source and no established method to reconstruct the profile of the Earth's interior compared with the medical computed tomography. As for the first difficulty we consider the neutrino pair beam proposed in Refs. [48,49]. It has been shown that pairs of neutrino and antineutrino can be produced from the partially stripped ions in circular motion at a larger rate than the current neutrino sources from pion and muon decays.

On the other hand, the second one is inherent in the tomography using the oscillation between flavor neutrinos. It has been shown [50–52] that the flavor oscillation probability with

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the density profile $\rho(x)$ is the same as that with $\rho(L-x)$ where $x=0$ or L is the production or detection position, if only two flavors of neutrinos are considered. In general the $\nu_\alpha \to \nu_\beta$ oscillation probability $P(\nu_\alpha \to \nu_\beta)$ with $\rho(x)$ is equal to $P(\nu_\beta \to \nu_\alpha)$ with $\rho(L-x)$ and an opposite sign of the Dirac-type CP-violating phase [52]. Because of the unitarity conditions $1=\sum_\alpha P(\nu_\alpha \to \nu_\beta)=\sum_\beta P(\nu_\alpha \to \nu_\beta)$ and the absence of the Dirac-type CP-violation in the two-flavor case $P(\nu_\alpha \to \nu_\beta)$ is invariant under $\rho(x) \to \rho(L-x)$. Even for the realistic three flavor case the invariance holds for the oscillations $\nu_e \to \nu_e$ and $\bar{\nu}_e \to \bar{\nu}_e$ [53,54] since they are independent on the Dirac-type phase.[1] In such cases unambiguous reconstruction of $\rho(x)$ is possible only if the profile has the symmetric property with $\rho(x)=\rho(L-x)$. Otherwise, there exist degenerate solutions of reconstruction. It has been, however, proposed that the difficulty can be avoided by using the transition probability of mass eigenstate to flavor eigenstate, which can be realized for the solar and supernova neutrinos [55,33]. Here we focus on the reconstruction of the symmetric density profile with $\rho(x)=\rho(L-x)$, and provide a useful procedure of its reconstruction. Procedures so far proposed are based on the χ^2 analysis (see, for example, Refs. [28,29]), the inverse Fourier transformation [33], and so on. The advantage of ours is that the reconstruction with a sufficient spatial resolution is possible even with a low numerical cost.

This letter is organized as follows: In section 2 we briefly review the neutrino oscillation in matter and present the analytic formula of the oscillation probability based on the perturbation of the matter effect, which will be used to reconstruct the density profile $\rho(x)$. In section 3 it is discussed the oscillation tomography using the neutrino pair beam. We show the possibility of the tomography under the ideal situation. It is then considered how to reconstruct $\rho(x)$ in section 4. Finally, our results are summarized in section 5.

2. Neutrino oscillation in matter

We begin with briefly reviewing the neutrino oscillation with matter effects [56–58]. The transition amplitude of the $\nu_\alpha \to \nu_\beta$ oscillation $(\alpha,\beta=e,\mu,\tau)$ at the distance x from neutrino source is denoted as

$$A_{\beta\alpha}(x)=\langle \nu_\beta | \nu_\alpha(x)\rangle\,, \tag{1}$$

where the initial condition is $|\nu_\alpha(0)\rangle=|\nu_\alpha\rangle$. It satisfies the following evolution equation.

$$i\frac{d}{dx}A_{\beta\alpha}(x)=\left[H_0^F+V^F(x)\right]_{\beta\gamma}A_{\gamma\alpha}(x)\,. \tag{2}$$

Here and hereafter we assume that all the neutrinos are ultrarelativistic. The free Hamiltonian in the basis of flavor neutrinos is H_0^F is given by

$$H_0^F=U\,H_0\,U^\dagger \quad \text{with} \quad H_0=\mathrm{diag}\left(\frac{m_1^2}{2E_\nu}\,,\ \frac{m_2^2}{2E_\nu}\,,\ \frac{m_3^2}{2E_\nu}\right)\,, \tag{3}$$

where E_ν is a neutrino energy, m_i $(i=1,2,3)$ is a neutrino mass eigenvalue and $U_{\alpha i}$ is the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix [59,60]. The effective potential in the flavor basis is given by

$$V^F(x)=\mathrm{diag}\left(\sqrt{2}\,G_F\,n_e(x)\,,\ 0\,,\ 0\right)\,, \tag{4}$$

where G_F is the Fermi constant and $n_e(x)$ is the number density of electrons at the distance x. By taking $n_p=n_n=n_e$ and $m_p=m_n$ in matter n_e can be written as

$$n_e(x)=\frac{\rho(x)}{2m_p}\,. \tag{5}$$

The matter density profile is denoted by $\rho(x)$. The oscillation probability measured by the detector at the distance $x=L$ is given by

$$P(\nu_\alpha \to \nu_\beta;E_\nu)=\left|A_{\beta\alpha}(L)\right|^2\,, \tag{6}$$

where the initial condition of the amplitude is $A_{\alpha\gamma}(0)=\delta_{\alpha\gamma}$. On the other hand, the amplitude of the antineutrino mode $\bar{A}_{\beta\alpha}(x)=\langle\bar{\nu}_\beta|\bar{\nu}_\alpha(x)\rangle$ is obtained by the replacements $U \to U^*$ and $V^F \to -V^F$.

The oscillation probability for a given matter density profile can be obtained by solving numerically Eq. (2). On the other hand, when the matter effect is sufficiently small, it is obtained analytically based on the perturbation theory [51,61]. We shall expand the amplitude as

$$A_{\beta\alpha}(x)=A_{\beta\alpha}^{(0)}(x)+A_{\beta\alpha}^{(1)}(x)+A_{\beta\alpha}^{(2)}(x)+\cdots\,, \tag{7}$$

where $A_{\beta\alpha}^{(n)}(x)$ is the n-th order correction of the matter effect. The explicit expressions up to the second order are

$$A_{\beta\alpha}^{(0)}(x)=U_{\beta j}U_{\alpha j}^*e^{-iE_jx}\,, \tag{8}$$

$$A_{\beta\alpha}^{(1)}(x)=-iU_{\beta j}U_{ej}^*U_{ek}U_{\alpha k}^*e^{-iE_jx}\int_0^x dx_1 e^{i(E_j-E_k)x_1}\,v(x_1)\,, \tag{9}$$

$$A_{\beta\alpha}^{(2)}(x)=-U_{\beta j}U_{ej}^*U_{ek}U_{ek}^*U_{el}U_{\alpha l}^*e^{-iE_jx}\times\int_0^x dx_1\int_0^{x_1}dx_2 e^{i(E_j-E_k)x_1}e^{i(E_k-E_l)x_2}\,v(x_1)\,v(x_2)\,, \tag{10}$$

where $E_i=\frac{m_i^2}{2E_\nu}$ and $v(x)$ is given by the density profile as

$$v(x)=\frac{G_F}{\sqrt{2}m_p}\rho(x)\,. \tag{11}$$

The oscillation probabilities at $x=L$ up to the second order are then given by

$$P^{(0)}(\nu_\alpha \to \nu_\beta;E_\nu)=|A_{\beta\alpha}^{(0)}(L)|^2\,, \tag{12}$$

$$P^{(1)}(\nu_\alpha \to \nu_\beta;E_\nu)=A_{\beta\alpha}^{(0)*}(L)A_{\beta\alpha}^{(1)}(L)+h.c.\,, \tag{13}$$

$$P^{(2)}(\nu_\alpha \to \nu_\beta;E_\nu)=|A_{\beta\alpha}^{(1)}(L)|^2+\left[A_{\beta\alpha}^{(0)*}(L)A_{\beta\alpha}^{(2)}(L)+h.c.\right]\,. \tag{14}$$

When there are only two flavors of neutrinos (ν_e and ν_μ), the mixing matrix is given by

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\,, \tag{15}$$

which leads to the zeroth and first order probabilities

$$P^{(0)}(\nu_e \to \nu_e;E_\nu)=1-\sin^2(2\theta)\sin^2\left(\frac{\Phi L}{2}\right)\,, \tag{16}$$

$$P^{(1)}(\nu_e \to \nu_e;E_\nu)=\frac{1}{2}\sin^2(2\theta)\cos(2\theta)\int_0^L dx_1\,v(x_1)\times\left[\sin(\Phi L)-\sin(\Phi x_1)-\sin(\Phi(L-x_1))\right]\,, \tag{17}$$

[1] This discussion cannot be applied to the case with sterile neutrinos [52].