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Modified Friedmann equations from Tsallis entropy

Ahmad Sheykhi [a,b,*]

[a] Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran
[b] Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran

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ABSTRACT

It was shown by Tsallis and Cirto that thermodynamical entropy of a gravitational system such as black hole must be generalized to the non-additive entropy, which is given by $S_h = \gamma A^\beta$, where A is the horizon area and β is the nonextensive parameter [1]. In this paper, by taking the entropy associated with the apparent horizon of the Friedmann–Robertson–Walker (FRW) Universe in the form of Tsallis entropy, and assuming the first law of thermodynamics, $dE = T_h dS_h + W dV$, holds on the apparent horizon, we are able to derive the corresponding Friedmann equations describing the dynamics of the universe with any spatial curvature. We also examine the time evolution of the total entropy and show that the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon. Then, modifying the emergence proposal of gravity proposed by Padmanabhan and calculating the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space, we again arrive at the modified Friedmann equation of the FRW Universe with any spatial curvature which is the same as one obtained from the first law of thermodynamics. We also study the cosmological consequences of Tsallis cosmology. Interestingly enough, we find that this model can explain simultaneously the late time acceleration in the universe filled with pressureless matter without invoking dark energy, as well as the early deceleration. Besides, the age problem can be circumvented automatically for an accelerated universe and is estimated larger than 3/2 age of the universe in standard cosmology. Taking $\beta = 2/5$, we find the age of the universe ranges as 13.12 Gyr $< t_0 <$ 16.32 Gyr, which is consistent with recent observations. Finally, using the Jeans's analysis, we comment, in brief, on the density perturbation in the context of Tsallis cosmology and found that the growth of energy differs compared to the standard cosmology.

1. Introduction

Although gravity is the most universal forces of nature, understanding its origin has been a mystery for a long time. Einstein believed that gravity is just the spacetime curvature and regarded it as an emergent phenomenon which describes the dynamics of spacetime. In the past decades a lot of attempts have been done to disclose the nature of gravity. A great step in this direction put forwarded by Jacobson [2] who studied thermodynamics of spacetime and showed explicitly that Einstein's equation of general relativity is just an equation of state for the spacetime. Combining the Clausius relation $\delta Q = T\delta S$, together with the entropy expression, he derived Einstein field equations. This derivation is of great importance because it confirms that the Einstein field equations is nothing but the first law of thermodynamics for the spacetime. Following Jacobson, a lot of studies have been carried out to disclose the deep connection between gravity and thermodynamics [3–5]. The studies were also generalized to the cosmological setups [6–14], where it has been shown that the Friedmann equation of Friedmann–Robertson–Walker (FRW) universe can be written in the form of the first law of thermodynamics on the apparent horizon. Although Jacobson's derivation is logically clear and theoretically sound, the statistical mechanical origin of the thermodynamic nature of general relativity remains obscure.

In 2010 Verlinde [15] put forwarded the next great step toward understanding the nature of gravity who claimed that gravity is not a fundamental force and can be interpreted as an entropic force caused by changes of entropy associated with the information on the holographic screen. Verlinde's proposal is based on two principles, namely the holographic principle and the equipartition law of energy. Using these principles he derived the Newton's law

* Correspondence to: Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran.
E-mail address: asheykhi@shirazu.ac.ir.

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of gravitation, the Poisson equation and in the relativistic regime the Einstein field equations [15] (see also [16]). The investigation on the entropic origin of gravity have been extended in different setups [17–24] and references therein). Although Verlinde's proposal has changed our understanding on the origin and nature of gravity, but it considers the gravitational field equations as the equations of emergent phenomenon and leave the spacetime as a pre-existed background geometric manifold. A new perspective towards emergence of spacetime dynamics was suggested in 2012 by Padmanabhan [25]. He argued that the spatial expansion of our Universe can be regarded as the consequence of emergence of space and *the cosmic space is emergent as the cosmic time progresses*. By calculating the difference between the number of degrees of freedom in the bulk and on the boundary, Padmanabhan [25] derived the Friedmann equation of the flat FRW Universe. This proposal was also applied for deriving the Friedmann equations of a higher dimensional FRW Universe in Einstein, Gauss–Bonnet and more general Lovelock cosmology [26,27]. By modification the Padmanabhan's proposal, one may extract the Friedmann equation of FRW Universe with any spatial curvature not only in Einstein gravity, but also in Gauss–Bonnet and more general Lovelock gravity [28]. The novel idea of Padmanabhan has got a lot of attentions in the literatures [29–35].

It is important to note that in order to rewrite the Friedmann equations, in any gravity theory, in the form of the first law of thermodynamics, $dE = T_h dS_h + WdV$, on the apparent horizon and vice versa, one should consider the entropy expression of the black hole in each gravity theory. The only change one should be done is replacing the black hole horizon radius r_+ by the apparent horizon radius $\tilde{r}_A$. However, the entropy expression associated with the black hole horizon get modified from the inclusion of quantum effects. Several types of quantum corrections to the area law have been introduced in the literatures, among them are logarithmic and power-law corrections. Logarithmic corrections, arises from the loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations [36–38]. The logarithmic term also appears in a model of entropic cosmology which unifies the inflation and late time acceleration [39]. Another form of correction to area law, namely the power-law correction, appears in dealing with the entanglement of quantum fields inside and outside the horizon [40–43]. Another modification for the area law of entropy comes from the Gibbs arguments who pointed out that in systems with divergency in the partition function, like gravitational system, the Boltzmann–Gibbs (BG) theory cannot be applied. As a result thermodynamical entropy of such nonstandard systems is not described by an additive entropy but must be generalized to the non-additive entropy [44]. Based on this, and using the statistical arguments, Tsallis and Cirto argued that the microscopic mathematical expression of the thermodynamical entropy of a black hole does not obey the area law and can be modified as [1],

$$S_h = \gamma A^{\beta}, \tag{1}$$

where A is the black hole horizon area, γ is an unknown constant and β known as Tsallis parameter or nonextensive parameter, which is a real parameter which quantifies the degree of nonextensivity [1]. It is obvious that the area law of entropy is restored for $\beta = 1$ and $\gamma = 1/(4L_p^2)$. Through this paper we set $k_B = 1 = c = \hbar$ for simplicity. In fact, at this limit, the power-law distribution of probability becomes useless, and the system is describable by the ordinary distribution of probability [1].

Let us briefly review the concept of nonextensive, or Tsallis entropy. In 1902 Gibbs pointed out that, in systems where the partition function diverges, the standard Boltzmann–Gibbs theory is not applicable, and large-scale gravitational systems are known to fall within this class. Tsallis generalized standard thermodynamics (which arises from the hypothesis of weak probabilistic correlations and their connection to ergodicity) to nonextensive one, which can be applied in all cases, and still possessing standard Boltzmann–Gibbs theory as a limit. Hence, the usual Boltzmann–Gibbs additive entropy must be generalized to the nonextensive, i.e. non-additive entropy (the entropy of the whole system is not necessarily the sum of the entropies of its sub-systems), which is named Tsallis entropy [45].

It is worth mentioning that in deriving Friedmann equations from the first law of thermodynamics, the entropy expression associated with the horizon plays a crucial role [12]. Thus, it is interesting to see how the Friedmann equation get modified if the entropy-area relation gets corrections by some reasons. Starting from the first law of thermodynamics at apparent horizon of a FRW universe, and assuming that the associated entropy with apparent horizon has a logarithmic quantum corrected relation, the modified Friedmann equations were derived in [46]. Also, taking the associated entropy with apparent horizon as the power-law-corrected relation, one is able to obtain the corrected Friedmann equation by using the first law of thermodynamics at the apparent horizon [47]. Besides, if thermodynamical interpretation of gravity near apparent horizon is generic feature, one should also not only check the first law of thermodynamics but also the generalized second law of thermodynamics. The latter is a universal principle governing the evolution of the total entropy of the Universe. In the context of the accelerating Universe, the generalized second law of thermodynamics has been explored in [48–51]. It should be noted that dark energy and a modified Friedmann equations in the context of Tsallis entropy and from different perspective, were first investigated in [45,52]. It was argued that Tsallis entropy parameter change the strength of the gravitational constant and consequently the energy density of the dark components of the universe, requiring more (less) dark energy to provide the observed late time universe acceleration [52]. They also explored some phenomenological aspects as well as some observational constraints from a modified Friedmann equations induced by Tsallis entropy [45,52]. In the context of the nonextensive Kaniadakis statistics [53], the Jeans length was investigated and the results were compared with the Jeans length obtained in the non-extensive Tsallis statistics [54]. Recently, modified cosmology through nonextensive Tsallis entropy have been investigated in [55]. It was shown that the universe exhibits the usual thermal history, with the sequence of matter and dark energy eras, and depending on the value of nonextensive parameter β the equation of state of dark energy can even cross the phantom-line [55]. In this work, we shall derive the modified Friedmann equation in a universe which its entropy is given by the nonextensive Tsallis entropy. Then, we investigate the cosmological implications of the obtained modified Friedmann equations in the matter and radiation dominated era. In our study, we do not need to invoke the dark energy component and the early deceleration as well as the late time acceleration of the universe expansion can be achieved in the presence of radiation and pressureless matter. Besides, we shall show that our model can solve the age problem of the universe. Our approach and the obtained Friedmann equations, from the first law of thermodynamics completely differ from those investigated in [45,52,55].

This paper is organized as follows. In the next section, we derive the modified Friedmann equations by applying the first law of thermodynamics, $dE = T_h dS_h + WdV$, at apparent horizon of a FRW universe and assuming the entropy associated with apparent horizon is in the form of Tsallis entropy (1). In section 3, we examine the generalized second law of thermodynamics for the total entropy including the corrected entropy-area relation together