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New generalized uncertainty principle from the doubly special relativity

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A R T I C L E I N F O

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A B S T R A C T

Based on the doubly special relativity we find a new type of generalized uncertainty principle (GUP) where the coordinate remains unaltered at the high energy while the momentum is deformed at the high energy so that it may be bounded from the above. For this GUP, we discuss some quantum mechanical problems in one dimension such as box problem, momentum wave function, and harmonic oscillator problem.

1. Introduction

As one method for curing some problems in quantum gravity, the generalization of the uncertainty relation has come, which is called a generalized uncertainty principle (GUP) [1–35]. There has been much development for the GUP formulation and GUP-corrected quantum systems. The generalized uncertainty principle (GUP) is given by the modified commutation relation

$$[X, P] = i(1 + \beta P^2), \tag{1}$$

where X and P is the position operator and the momentum operator, respectively, and β is a small parameter given by $\beta = \beta_0/M_{pl}c^2$, and M_{pl} is the Planck mass and β_0 is of order the unity, and we set $\hbar = 1$. The GUP guarantees the non-zero minimal length and is related to the quantization of gravity.

More generally, the modified commutation relation can be written as [23–25,33–35]

$$[X, P] = iF(P), \tag{2}$$

where $F(P)$ is called a GUP deformation function which reduces to 1 when the GUP effect is ignored. From now on we will call the eq. (2) a generalized GUP. Here, (X, P) implies the coordinate and momentum at the high energy while (x, p) is the coordinate and momentum at the low energy where x and p is defined through $[x, p] = i$. At the high energy the coordinate and momentum is assumed to be deformed through

$$x \to X = x, \quad p \to P = f(p) \tag{3}$$

The eq. (3) implies that the position remains undeformed at the high energy while the momentum is deformed at the high energy. For the GUP (1), we have

$$P = \frac{1}{\sqrt{\beta}} \tan(\sqrt{\beta} p) \tag{4}$$

In this case the momentum at the high energy is not bounded, $-\infty < P < \infty$. This seems strange because the momentum at the high energy should be bounded if we consider the doubly special relativity (DSR) [36–40]. Indeed DSR says that the momentum has the maximum called a Planck momentum which is another invariant in DSR.

Therefore, in order to construct the new GUP with both minimum length and maximum momentum, we should find the mapping (or deformation) $X = x$, $P = f(p)$ with $f(\pm\infty) = \pm\kappa$. Thus, from this map, the momentum operator has the maximum value (Planck momentum κ).

In this paper we are to find a new type of GUP where the coordinate remains unaltered at the high energy while the momentum is deformed at the high energy so that it may be bounded from the above. Our GUP model comes from the concept of DSR. This paper is organized as follows: In section 2 we discuss the brief review of representation of generalized GUP. In section 3 we discuss the new GUP from the concept of DSR. In section 4 we discuss the momentum wave function in a position representation. In section 5 we discuss one dimensional box problem. In section 6 we discuss harmonic oscillator problem.

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2. Brief review of representation of generalized GUP

Now let us reconsider the generalized GUP

$$[X,P]=iF(P). \tag{5}$$

For this commutation relation we have two representations.

2.1. Deformed momentum representation

The deformed momentum representation for the algebra (5) is

$$X=iF(p)\frac{\partial}{\partial p},\quad P=p \tag{6}$$

The momentum representation acts on the square integrable functions $\Phi(p)\in\mathcal{L}^2\left(-A,A;\frac{dp}{F(p)}\right)$ with $\phi(\pm A)=0$ and the norm of ϕ is given by

$$||\Phi||^2=\int_{-\infty}^{\infty}\frac{dp}{F(p)}|\Phi(p)|^2 \tag{7}$$

For the standard GUP (1) we have $A=\infty$. The Schrödinger equation reads

$$\left[\frac{p^2}{2m}+V\left(iF(p)\frac{\partial}{\partial p}\right)\right]\Phi(p)=E\Phi(p) \tag{8}$$

2.2. Position representation

The position representation for the algebra (5) is

$$X=x,\quad P=f(p)=f\left(\frac{1}{i}\partial_x\right) \tag{9}$$

where the function f is obtained from

$$p=\int^{P}\frac{dP}{F(P)} \tag{10}$$

The position representation acts on the square integrable functions $\psi(x)\in\mathcal{L}^2(-\infty,\infty;dx)$ and the norm of ψ is given by

$$||\psi||^2=\int_{-\infty}^{\infty}dx|\psi(x)|^2 \tag{11}$$

The Schrödinger equation reads

$$\left[\frac{1}{2m}\left(f\left(\frac{1}{i}\partial_x\right)\right)^2+V(x)\right]\psi(x)=E\psi(x) \tag{12}$$

2.3. Undeformed momentum representation

The undeformed momentum representation for the algebra (5) is obtained from the position representation with replacing $x=i\partial_p$, as

$$X=x=i\partial_p,\quad P=f(p)=f(p) \tag{13}$$

The undeformed momentum representation acts on the square integrable functions $\phi(p)\in\mathcal{L}^2(-\infty,\infty;dp)$ and the norm of ψ is given by

$$||\phi||^2=\int_{-\infty}^{\infty}dp|\phi(p)|^2 \tag{14}$$

The Schrödinger equation reads

$$\left[\frac{1}{2m}(f(p))^2+V\left(i\partial_p\right)\right]\phi(p)=E\phi(p) \tag{15}$$

3. New GUP from the concept of DSR

In the DSR, the energy-momentum relation is deformed into

$$E^2=p_0^2+m^2+h(|p_0|,\kappa), \tag{16}$$

where we demand that

$$\lim_{\kappa\to\infty}h(|p_0|,\kappa)=0 \tag{17}$$

and p_0 means that it is not a operator but a number, and we set $c=1$. The undeformed momentum p_0 (momentum at the low energy) can be related to the deformed momentum P_0 (momentum at the high energy) as follows:

$$P_0=f(p_0) \tag{18}$$

If we choose

$$P_0=f(p_0)=\frac{p_0}{1+\frac{|p_0|}{\kappa}}, \tag{19}$$

we have the modified dispersion relation

$$E^2=P_0^2+m^2=\left[\frac{p_0}{1+\frac{|p_0|}{\kappa}}\right]^2+m^2 \tag{20}$$

or

$$E^2=p_0^2+m^2-2\frac{|p_0|}{\kappa}p^2+3\frac{p^4}{\kappa^2}+\cdots \tag{21}$$

We can easily check that the choice (19) obeys

$$\lim_{p_0\to\pm\infty}P_0=\lim_{p_0\to\pm\infty}f(p_0)=\pm\kappa \tag{22}$$

which gives $|P_0|\le\kappa$.

Based on the DSR, we consider the following relation for the momentum operators

$$P=\frac{p}{1+\frac{|p|}{\kappa}}, \tag{23}$$

where $|p|$ is the magnitude of undeformed momentum operator p, or $|p|=\sqrt{p^2}$. The inverse transformation is

$$p=\frac{P}{1-\frac{|P|}{\kappa}} \tag{24}$$

From the requirement $|p|\ge 0$ we get $|P|\le\kappa$ which gives the upper bound for the momentum at the high energy. The limit $p\to\pm\infty$ corresponds to $P=\pm\kappa$, which implies that there exists the maximum momentum (Planck momentum) in our model. The eq. (23) gives the following commutation relation

$$[X,P]=i\left(1-\frac{|P|}{\kappa}\right)^2, \tag{25}$$

which gives

$$\Delta X\Delta P\ge\frac{1}{2}\left(1-2\frac{\langle|P|\rangle}{\kappa}+\frac{1}{\kappa^2}(\Delta P)^2\right), \tag{26}$$

where we set $\langle P\rangle=0$. From now on we call the above GUP the DSR-GUP. The eq. (26) gives the minimal length

$$(\Delta X)_{min}=\frac{1}{\kappa}\sqrt{1-2\frac{\langle|P|\rangle}{\kappa}} \tag{27}$$

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