Physics Letters B 785 (2018) 296–300

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Holographic quantum critical conductivity from higher derivative electrodynamics

Jian-Pin Wu [a,b]

[a] Center for Gravitation and Cosmology, College of Physical Science and Technology, Yangzhou University, Yangzhou 225009, China
[b] Institute of Gravitation and Cosmology, Department of Physics, School of Mathematics and Physics, Bohai University, Jinzhou 121013, China

ARTICLE INFO

Article history:
Received 22 July 2018
Received in revised form 19 August 2018
Accepted 3 September 2018
Available online 5 September 2018
Editor: N. Lambert

ABSTRACT

We study the conductivity from higher derivative electrodynamics in a holographic quantum critical phase (QCP). Two key features of this model are observed. First, a rescaling for the Euclidean frequency by a constant is needed when fitting the quantum Monte Carlo (QMC) data for the $O(2)$ QCP. We conclude that it is a common characteristic of the higher derivative electrodynamics. Second, both the Drude-like peak at low frequency and the pronounced peak can simultaneously emerge. They are more evident for the relevant operators than for the irrelevant operators. In addition, our result also further confirms that the conductivity for the $O(2)$ QCP is particle-like but not vortex-like. Finally, the electromagnetic (EM) duality is briefly discussed. The largest discrepancies of the particle–vortex duality in the boundary theory appear at the low frequency and the particle–vortex duality holds more well for the irrelevant operator than for the relevant operator.

1. Introduction

Quantum critical (QC) system, which includes quantum critical phase transition (QCPT) and quantum critical phase (QCP), is a long-standing important issue in condensed matter physics [1]. Some of the best understood examples are described by strongly interacting conformal field theory (CFT) at low energy. A canonical example is the superfluid-insulator QCP described by the boson Hubbard model.

The real-time dynamics, especially the frequency-dependent conductivity $\sigma(\omega)$, at finite temperature is a central and challenging subject in QC physics [2]. Because of the strongly correlated nature of QC system, the conventional perturbative methods in traditional quantum field theory (QFT) unfortunately lose its power in studying the dynamics. The novel non-perturbative techniques and methods are called for.

AdS/CFT correspondence [3–6] provides a powerful tool in dealing with the real-time dynamics of the strongly interacting QC system lacking quasi-particles. References [8,9,7] construct holographic models based on the Maxwell-Weyl system in Schwarzschild-AdS (SS-AdS) to study QC physics, in particular the dynamical conductivity $\sigma(\omega)$. By combining high precision quantum Monte Carlo (QMC) results [7,8,10] for the dynamical conductivity in the $O(2)$ QCP with that from holography [8,9,7], they build a quantitative description of the dynamics of QC systems lacking quasi-particles and find that the dynamical conductivity for the $O(2)$ QCP is particle-like but not vortex-like, which resolved the puzzle of $O(2)$ QCP.

Further studies find that the relevant scalar operator plays a key role in the dynamics of the QC systems [7]. However, the scalar field in the bulk introduced in [7], which is dual to the relevant operator in the boundary field theory, is not a dynamical field. To overcome this shortcoming, references [11,12] construct a novel neutral scalar hair black brane by coupling Weyl tensor with neutral scalar field, which provides a framework to describe QC dynamics and the one away from QCP. In particular, the relevant operator of this model acquires a thermal expectation value and we can study the dynamical conductivity for a wide range of conformal dimensions Δ.

However, until now AdS/CFT is best understood only at large-N limit [3–6]. Therefore, it is important to study the universality and the speciality of the dynamics of the QC systems. To this end, here we extend the studies in [11,12] to include a higher derivative term by incorporating a interaction between gauge field and Weyl tensor and explore the generic and special properties of the holographic QC dynamics.

E-mail address: jianpinwu@yzu.edu.cn.

https://doi.org/10.1016/j.physletb.2018.09.003

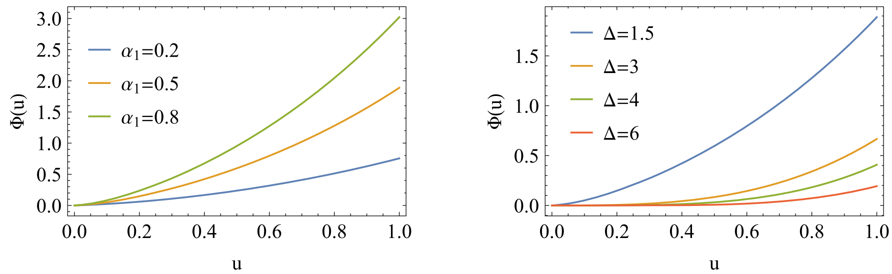

Fig. 1. Plots of $\Phi(u)$ as the function of u for sample Δ and α_1. Left plot is for $\Delta = 1.5$ and different α_1 and right plot is for $\alpha_1 = 0.5$ and different Δ.

2. Holographic framework

We start with the following SS-AdS black brane

$$ds^2 = \frac{r_0^2}{L^2u^2}\Big(-f(u)dt^2 + dx^2 + dy^2\Big) + \frac{L^2}{u^2 f(u)}du^2\,, \tag{1a}$$

$$f(u) = (1-u)p(u)\,, \qquad p(u) = u^2 + u + 1\,. \tag{1b}$$

$u = 0$ is the asymptotically AdS boundary while the horizon locates at $u = 1$. The Hawking temperature of this system is

$$T = \frac{3r_0}{4\pi L^2}\,. \tag{2}$$

We study the following bulk action including a massless gauge field A_μ, and a scalar field Φ

$$S_\Phi = -\frac{1}{2l_p^2}\int d^4x\sqrt{-g}\Big[(\nabla_\mu\Phi)^2 + m^2\Phi^2 - 2\alpha_1 L^2\Phi C^2\Big]\,, \tag{3a}$$

$$S_A = -\int d^4x\sqrt{-g}\Big(\frac{1}{8g_F^2}F_{\mu\nu}X^{\mu\nu\rho\sigma}F_{\rho\sigma}\Big)\,. \tag{3b}$$

The scalar field Φ in bulk gravity is dual to the scalar operator $\mathcal{O}$ with conformal dimension $\Delta = \frac{1}{2}(3 \pm \sqrt{9 + 4m^2L^2})$ in boundary CFT. In the action S_A, $F = dA$ is the curvature of gauge field A and the tensor X is

$$X_{\mu\nu}{}^{\rho\sigma} = (1 + \alpha_2\Phi)I_{\mu\nu}{}^{\rho\sigma} - 8\gamma\Phi L^4 C_{\mu\nu}{}^{\rho\sigma}\,, \tag{4}$$

where $I_{\mu\nu}{}^{\rho\sigma} = \delta_\mu{}^\rho\delta_\nu{}^\sigma - \delta_\mu{}^\sigma\delta_\nu{}^\rho$ is an identity matrix. In the above equations (3), we have introduced the factors of l_p and L so that the coupling parameters g_F, $\alpha_{1,2}$, γ, and the scalar field Φ are dimensionless. Without loss of generality, we set $l_p^2 = 1/2$, $g_F = 1$ and $L = 1$ in what follows. Comparing with [11], we introduce a new interaction term in Eq. (4) that coupling among the Weyl tensor, gauge field and scalar field.

The black brane geometry (1) describes a thermal state in the dual boundary CFT. Following the strategy in [11], we introduce an interaction term between the scalar field Φ and the Weyl tensor such that the scalar field have a nontrivial profile in the black brane background, which corresponds to a nonvanishing thermal expectation value of scalar operator in boundary theory.

From the action (3), we obtain the EOMs for the scalar field and gauge field as

$$(\nabla^2 - m^2)\Phi + \alpha_1 L^2 C^2 - \frac{1}{16}(\alpha_2 I_{\mu\nu}{}^{\rho\sigma} - 8\gamma C_{\mu\nu}{}^{\rho\sigma})F^{\mu\nu}F_{\rho\sigma} = 0\,, \tag{5a}$$

$$\nabla_\nu(X^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0\,. \tag{5b}$$

Since here we consider a thermal state, which described by the neutral black brane background. In this case, the background gauge field is zero. Therefore, Eq. (5a) reduces to

$$(\nabla^2 - m^2)\Phi + \alpha_1 L^2 C^2 = 0\,. \tag{6}$$

The above EOM determines the profile of the scalar field.

Since the Weyl tensor vanishes in the AdS boundary, the asymptotic behavior of $\Phi(u)$ is the same as that without the α_1 coupling term, which behaves

$$\Phi(u) = \Phi_0 u^{3-\Delta} + \Phi_1 u^\Delta\,. \tag{7}$$

We identify Φ_0 as the source, which corresponds to the coupling of the boundary QFT and deforms it, and Φ_1 as the expectation. The conformal dimension Δ is constrained in $\Delta \geq 1/2$ such that the dual CFTs are unitary [13]. When $\Phi_0 = 0$, the dual theory is the QCP [11]. If we tune Φ_0 nonzero, the dual theory is away from QCP [11]. In this paper, we only focus on the case of $\Phi_0 = 0$.

Combining the falling of $\Phi(u)$ (Eq. (7)) at the boundary $u \to 0$ and the regular requirement of $\Phi(u)$ at the horizon, we can numerically solve this EOM and show the profile of scalar field for sample Δ and α_1 in Fig. 1. From this figure, we can see that the value of $\Phi(u)$ at the horizon increases with the increase of α_1 for fixed Δ. While for fixed α_1, the value of $\Phi(u)$ at the horizon increases with the decrease of Δ.

3. Holographic conductivity

To calculate the frequency-dependent conductivity, we turn on the perturbation of the gauge field at zero momentum along y direction in Fourier space as $A_y \sim a_y(u)e^{-i\omega t}$ and the EOM for the gauge field (5b) can be explicitly wrote down as

$$a_y'' + a_y'\left(\frac{f'}{f} + \frac{3\alpha_2\Phi' - 2\gamma u\left(f^{(3)}u\Phi + f''\left(u\Phi' + 2\Phi\right)\right)}{3\alpha_2\Phi - 2\gamma u^2\Phi f'' + 3}\right) + \frac{\omega^2 a_y}{f^2} = 0\,. \tag{8}$$

And then, the conductivity is given by

$$\sigma(\omega) = \frac{\partial_u a_y}{i\omega a_y}\Big|_{u\to 0}\,. \tag{9}$$

Imposing the ingoing boundary condition at the horizon, we can numerically solve the EOM (8) and read off the conductivity by Eq. (9).

Subsequently, we shall study the conductivity at QCP by tuning $\Phi_0 = 0$. We mainly study the properties of the conductivity from higher derivative electrodynamics in the holographic framework described in the last section.

Previously, in [11], only when the α_2 term survives, i.e., turning off γ here, it has been shown that the conductivity for Euclidean frequency can be fitted very well for $\Omega > 2\pi T$ to the QMC data for the $O(2)$ QCP [7,8,10]. In fact, before that, the authors in [8] have found that the QMC data for the $O(2)$ QCP can also be fitted in a simple holographic model in which only the coupling term