Physics Letters B 785 (2018) 362–371

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Backreacting holographic superconductors from the coupling of a scalar field to the Einstein tensor

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A R T I C L E I N F O

Article history:
Received 16 June 2018
Received in revised form 23 August 2018
Accepted 30 August 2018
Available online 3 September 2018
Editor: N. Lambert

A B S T R A C T

We investigate the properties of the backreacting holographic superconductors from the coupling of a scalar field to the Einstein tensor in the background of a d-dimensional AdS black hole. Imposing the Dirichlet boundary condition of the trial function without the Neumann boundary conditions, we improve the analytical Sturm–Liouville method with an iterative procedure to explore the pure effect of the Einstein tensor on the holographic superconductors and find that the Einstein tensor hinders the condensate of the scalar field but does not affect the critical phenomena. Our analytical findings are in very good agreement with the numerical results from the "marginally stable modes" method, which implies that the Sturm–Liouville method is still powerful to study the holographic superconductors from the coupling of a scalar field to the Einstein tensor even if we consider the backreactions.

1. Introduction

It is well known that the superconductivity is one of the most remarkable phenomena observed in physics in the 20th century [1]. However, the core mechanism of the high-temperature superconductor systems, which can not be described by the usual Bardeen–Cooper–Schrieffer (BCS) theory [2], is still one of the unsolved mysteries in theoretical physics so far. Interestingly, it was suggested that it is logical to investigate the properties of high temperature superconductors on the boundary of spacetime by considering the classical general relativity in one higher dimension with the help of the Anti-de Sitter/conformal field theories (AdS/CFT) correspondence [3–5]. In the probe limit, Gubser observed that the spontaneous $U(1)$ symmetry breaking by bulk black holes can be used to construct gravitational dual of the transition from normal state to superconducting state [6], and Hartnoll et al. found that the properties of a $(2+1)$-dimensional superconductor can indeed be reproduced in the $(3+1)$-dimensional holographic dual model based on the framework of usual Maxwell electrodynamics [7]. Extending the investigation to the so-called holographic superconductor models away from the probe limit, i.e., taking the backreactions of the spacetime into account, the authors of Ref. [8] showed that even the uncharged scalar field can form a condensate in the $(2+1)$-dimensional holographic superconductor model. Along this line, there has been accumulated interest in studying the effects of the backreaction on the holographic s-wave [9–41] p-wave [42–49] and d-wave [50] dual models. Reviews of the holographic superconductors can be found in Refs. [51–54].

Most of the aforementioned works on the gravitational dual models focus on the superconductors without an impurity. As a matter of fact, to study the effect of impurities is often important since their presence can drastically change the physical properties of the superconductors in condensed matter physics [55]. According to the AdS/CFT duality, Ishii and Sin investigated the impurity effect in a holographic superconductor by turning on a coupling between the gauge field and a new massive gauge field, and found that the mass gap in the optical conductivity disappears when the coupling is sufficiently large [56]. Zeng and Zhang studied the single normal impurity effect in a superconductor by using the holographic approach, which showed that the critical temperature of the host superconductor decreases as the size of the impurity increases and the phase transition at the critical impurity strength (or the critical temperature) is of zeroth order [57]. Fang et al. extended the study to the Fermionic phase transition induced by the effective impurity in holography and obtained a phase diagram in (α, T) plane separating the Fermi liquid phase and the non-Fermi liquid phase [58]. More recently, Kuang and Papantonopoulos built a holographic superconductor with a scalar field coupled kinematically to the Einstein tensor and observed that, as the strength of

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https://doi.org/10.1016/j.physletb.2018.08.069

the coupling increases, the critical temperature below which the scalar field condenses is lowering, the condensation gap decreases faster than the temperature, the width of the condensation gap is not proportional to the size of the condensate and at low temperatures the condensation gap tends to zero for the strong coupling [59]. Obviously, these effects suggest that the derivative coupling in the gravity bulk can have a dual interpretation on the boundary corresponding to impurities concentrations in a real material. Note that they concentrated on the probe limit where the backreaction of matter fields on the spacetime metric is neglected. Thus, in this work we will extend their interesting model to the case away from the probe limit and explore the effect of the Einstein tensor on the holographic superconductors with backreactions. In addition, we will compare the result in five dimensions with that in four dimensions and present an analysis of the effect the extra dimension has on the scalar condensation formation. In the calculation, we first use the Sturm–Liouville eigenvalue problem [60,61] to analytically study the holographic superconductor phase transition, and then count on the "marginally stable modes" method [6,62] to numerically confirm the analytical findings and verify the effectiveness of the Sturm–Liouville method.

The organization of the work is as follows. In Sec. 2, we will introduce the backreacting holographic superconductor models from the coupling of a scalar field to the Einstein tensor in the d-dimensional AdS black hole background. In Sec. 3 we will give an analytical investigation of the holographic superconductors by using the Sturm–Liouville method. In Sec. 4 we will give a numerical investigation of the holographic superconductors by using the "marginally stable modes" method. We will summarize our results in the last section.

2. Description of the holographic dual system

The general action describing a charged, complex scalar field coupled to the Einstein tensor $G^{\mu\nu}$ in the d-dimensional Einstein–Maxwell action with negative cosmological constant $\Lambda=-(d-1)(d-2)/(2L^2)$ is of the form

$$S=\int d^dx\sqrt{-g}\left[\frac{1}{2\kappa^2}(R-2\Lambda)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-(g^{\mu\nu}+\eta G^{\mu\nu})D_\mu\psi(D_\nu\psi)^*-m^2|\psi|^2\right],\qquad(1)$$

with $D_\mu=\nabla_\mu-iqA_\mu$. Here $\kappa^2=8\pi G_d$ represents the gravitational constant, $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$ is the Maxwell field strength, and ψ denotes the scalar field with the charge q and mass m. When the coupling parameter $\eta\to0$, our model reduces to the standard holographic superconductors with backreactions investigated in [8,9]. It should be noted that we can rescale the bulk fields ψ and A_μ as ψ/q and A_μ/q in order to put the factor $1/q^2$ as the backreaction parameter for the matter fields. So the probe limit can be obtained safely if $\kappa^2/q^2\to0$. Without loss of generality, we can set $q=1$ and keep κ^2 finite when we take the backreaction into account, just as in Refs. [9–13,46].

To go beyond the probe limit, we adopt the metric ansatz for the black hole with the curvature $k=0$ as

$$ds^2=-f(r)e^{-\chi(r)}dt^2+\frac{dr^2}{f(r)}+r^2h_{ij}dx^idx^j,\qquad(2)$$

where f and χ are functions of r only, $h_{ij}dx^idx^j$ represents the line element of a $(d-2)$-dimensional hypersurface. Obviously, the Hawking temperature of this d-dimensional black hole, which will be interpreted as the temperature of the CFT, can be given by

$$T_H=\frac{f'(r_+)e^{-\chi(r_+)/2}}{4\pi},\qquad(3)$$

where the prime denotes a derivative with respect to r, and the black hole horizon r_+ is determined by $f(r_+)=0$. For the considered ansatz (2), the nonzero components of the Einstein tensor $G^{\mu\nu}$ are

$$G^{tt}=-\frac{(d-2)e^\chi}{2r^2}\left[(d-3)+\frac{rf'}{f}\right],$$
$$G^{rr}=\frac{(d-2)f^2}{2r^2}\left[(d-3)+\frac{rf'}{f}-r\chi'\right],$$
$$G^{xx}=G^{yy}=\cdots=\frac{1}{4r^3}\left\{f'\left[4(d-3)-3r\chi'\right]+2rf''+f\left[\frac{2(d-3)(d-4)}{r}-2(d-3)\chi'+r\chi'^2-2r\chi''\right]\right\}.\qquad(4)$$

For the scalar field and electromagnetic field, we will take $\psi=|\psi|$, $A_t=\phi$ where ψ, ϕ are both real functions of r only. Thus, from the action (1) we can give the equations of motion for the metric functions $f(r)$ and $\chi(r)$

$$\left[1+\eta\kappa^2\left(\frac{e^\chi\phi^2\psi^2}{f}-3f\psi'^2\right)\right]\chi'+\frac{4\kappa^2r}{d-2}\left\{\psi'^2+\frac{e^\chi\phi^2\psi^2}{f^2}+\frac{(d-2)\eta}{2r}\left[\frac{(d-3)f}{r}\left(\psi'^2+\frac{e^\chi\phi^2\psi^2}{f^2}\right)+\frac{2e^\chi\phi\psi(\phi\psi)'}{f}-2f\psi'\psi''\right]\right\}=0,\qquad(5)$$

$$\left[1+\eta\kappa^2\left(\frac{e^\chi\phi^2\psi^2}{f}-3f\psi'^2\right)\right]f'-\left[\frac{(d-1)r}{L^2}-\frac{(d-3)f}{r}\right]+\frac{2\kappa^2r}{d-2}\left\{m^2\psi^2+\frac{e^\chi\phi'^2}{2}+f\left(\psi'^2+\frac{e^\chi\phi^2\psi^2}{f^2}\right)+\frac{(d-2)\eta}{2r}\left[\frac{(d-3)e^\chi\phi^2\psi^2}{r}-\frac{(d-3)f^2\psi'^2}{r}-4f^2\psi'\psi''\right]\right\}=0,\qquad(6)$$

and for the matter fields $\phi(r)$ and $\psi(r)$

$$\phi''+\left(\frac{d-2}{r}+\frac{\chi'}{2}\right)\phi'-\frac{2\psi^2}{f}\left[1+\frac{(d-2)\eta f}{2r}\left(\frac{d-3}{r}+\frac{f'}{f}\right)\right]\phi=0,\qquad(7)$$

$$\left\{1+\frac{(d-2)\eta}{2r}\left[\frac{(d-3)f}{r}+f'-f\chi'\right]\right\}\psi''+\left\{\left(\frac{d-2}{r}+\frac{f'}{f}-\frac{\chi'}{2}\right)+\frac{(d-2)\eta}{2r}\left[f''+\frac{3(d-3)f'}{r}+\frac{f'^2}{f}+\frac{f\chi'^2}{2}-\frac{3(d-3)f\chi'}{2r}-\frac{5f'\chi'}{2}-f\chi''+\frac{(d-3)(d-4)f}{r^2}\right]\right\}\psi'+\left\{\frac{e^\chi\phi^2}{f^2}\left[1+\frac{(d-2)(d-3)\eta f}{2r^2}+\frac{(d-2)\eta f'}{2r}\right]-\frac{m^2}{f}\right\}\psi=0,\qquad(8)$$

where the prime denotes a derivative with respect to r.