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Holographic mutual and tripartite information in a symmetry breaking quench

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ABSTRACT

We study the time evolution of holographic mutual and tripartite information for a zero temperature CFT, derives to a non-relativistic thermal Lifshitz field theory by a quantum quench. We observe that the symmetry breaking does not play any role in the phase space, phase of parameters of sub-systems, and the length of disentangling transition. Nevertheless, mutual and tripartite information indeed depend on the rate of symmetry breaking. We also find that for large enough values of δt the quantity $t_{eq}\delta t^{-1}$, where δt and t_{eq} are injection time and equilibration time respectively, behaves adiabatically, *i.e.* its value is independent of length of separation between sub-systems. We also show that tripartite information is always non-positive during the process indicates that mutual information is monogamous.

1. Introduction and results

The gauge/gravity duality [1,2], as the most concrete realization of holographic principle, has been of interest to physicist over the years [2–6]. The basic idea is that a gravitational theory defined on a $d+1$ dimensional background, the bulk, is equivalent to a gauge theory defined on a d dimensional spacetime that forms the bulk's boundary. This correspondence is also a weak/strong duality which has been a useful and powerful tool to study the strongly coupled field theories by gravitational description [1,2,7,8]. Surprisingly, it is extended to the time dependent cases and therefore is appropriate to study the non-equilibrium phenomenon. Various areas raging from Relativistic Heavy Ion Collider to condensed matter physics are tried to explain with this duality (for a review see [9–12]).

Entanglement entropy is one of the most intriguing non-local quantities which measures the quantum entanglement between two sub-systems of a given system. It can be also used to classify the various quantum phase transitions and critical points [13–15]. Since the quantum field theories have infinitely degrees of freedom, the entanglement entropy is divergent. Thus, it is scheme-dependent quantity and needs to be regulated. It has been shown that the leading divergence term is proportional to the area of the entangling surface (for $d > 2$) [16,17]

$$S_{EE} \propto \frac{Area}{\epsilon^{d-2}}, \tag{1}$$

where ϵ is the UV cut-off in quantum field theories. This is called the area law (see also [18,19]). Note that cut-off dependence of the entanglement entropy makes it to be a non-universal quantity.

Due to the UV divergence structure of entanglement entropy, it is natural to introduce an appropriate quantity called mutual information which is an important concept in information theory and has more advantages than the entanglement entropy. It is a finite, positive, semi-definite quantity which measures the total correlation between the two sub-systems A and B [20]. The tripartite information is another useful quantity in this context which is defined for a system consisting of three spatial regions and measures the extensivity of the mutual information. It is also free of divergence and can take any value depending on the underlying field theory. In spite of the mutual information, tripartite information is finite even when the regions share boundaries [21].

To understand AdS/CFT (for a review see [8]), a particular case of gauge/gravity duality where the gravity lives in a background with a negative cosmological constant, it seems highly important to study how the information in the CFT is encoded in the gravity theory. Since the amount of the information of a sub-system A can be measured by the entanglement entropy of that sub-system, it seems natural to ask how one can calculate this in the gravity

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side. In [22,23], by applying AdS/CFT correspondence, the authors showed that the entanglement entropy of a region A in a CFT is proportional to the area of a surface which has the minimum area among surfaces whose boundaries coincide with the boundary of the region A which is known as Ryu–Takayanagi (RT) prescription. Since both mutual information and tripartite information are combinations of entanglement entropy they can be then calculated by RT prescription. Consequently, if one would like to calculate the amount of the correlation between two sub-system A and B, then mutual information is the quantity needs to be computed and the tripartite information is a quantity to study the degree of the extensivity of the mutual information.

We are interested to study equilibration process by mutual information and to figure out how symmetry breaking can effect the evolution of them. Besides, we want to investigate effect of symmetry breaking on the monogamy of the tripartite information. To do so, we study the time evolution of the holographic mutual and tripartite information of a strongly coupled CFT (initial state) which is derived to a non-relativistic fixed point with Lifshitz scaling (final state). On the gravity side, this non-equilibrium dynamics is equivalent to a special geometry interpolating between a pure AdS at past infinity and an asymptotically Lifshitz black hole at future infinity. We find the following interesting results corresponding to the mutual and tripartite information of the underlying background.

- The symmetry breaking has no effect on the phase diagram of the sub-systems and the separation length of them. While it shifts mutual and tripartite information.
- For slow quenches the quantity $t_{eq}\delta t^{-1}$, where δt and t_{eq} are injection time and equilibration time respectively, behaves universally. In other words, the length of separation between subsystems does not affect this quantity.
- The non-equilibrium dynamics following the breaking of the relativistic scaling symmetry leads to the more correlation between two sub-systems. Namely, the less symmetry, the greater correlation.
- For slow quenches the mutual information approaches the adiabatic regime in the final state, *i.e.* there is no dependence on the separation length between two sub-systems.
- Mutual information does undergo a disentangling transition, for a given value of the separation length between two subsystems, beyond which it is identically zero. Moreover, the separation length of disentangling transition corresponding to the final state is bigger than that of the initial state.
- There is a specific regime of the parameters, small enough of the length of two sub-systems and their separation length, in the phase space diagram of two sub-systems where the mutual information is independent of the time evolution.
- The tripartite information is always non-positive during the symmetry breaking quench. Therefore, mutual information is monogamous.

2. Review on background

The gauge/gravity duality [1,2] provides a wide range of domain to study strongly coupled quantum field theories whose dual are the gravitational theories in one higher dimension. This conjectured duality has been used to explore applications in condensed matter physics and quantum chromodynamics (for a review see [24]). In the context of condensed matter, there are quantum systems exhibiting a non-relativistic scaling, which refers to as Lifshitz scaling in the literature, of the following form in $d+1$ dimensions

$$(t, x) \longrightarrow (\lambda^z t, \lambda x^i), \tag{2}$$

where z is a dynamical critical exponent governing the anisotropy between spatial and temporal scaling and x^i $(i = 1, 2, ...d)$ denotes the spatial coordinates. The gauge/gravity logic suggests that one can look for a background metric in one higher dimension than the field theory whose symmetries match with a field theory living on the boundary. In our case the following Lifshitz geometry was proposed in [25,26] as a candidate background for the holographic dual of such a non-relativistic theory

$$ds^2 = -\frac{r^{2z}}{L^{2z}}dt^2 + \frac{r^2}{L^2}d\mathbf{x}^2 + \frac{r^2}{L^2}dr^2, \tag{3}$$

where $l_{AdS} \equiv L(z=1)$, z can take any positive number and the scale transformation acts as (2) along with $r \to r\lambda^{-1}$. The case $z=1$ is the famous Anti-de Sitter spacetime whose symmetry, and its dual scale-invariant theory, is substantially enhanced. AdS geometry is a vacuum solution to a simple $d+1$ dimensional theory of gravity, namely general relativity with a negative cosmological constant

$$S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-g}(R + \frac{d(d-1)}{L^2}), \tag{4}$$

where G_{d+1} is Newton constant and R is the Ricci scalar. Solutions with Lifshitz isometries were first presented in [26]. Einstein gravity with a negative cosmological constant alone does not support the geometry and hence general relativity must be coupled with some matter content. There are many models have been proposed in the literature to reach this Lifshitz solution such as, Einstein–Proca, Einstein–Maxwel–Dilaton and Einstein-p form actions [26–29] or using the nonrelativistic gravity theory of Horava–Lifshitz [31]. Here we consider a model involving gravity with negative cosmological constant and a massive gauge field whose action has the following form [29]

$$S = \frac{1}{16\pi G_{d+1}} \int d^{d+1}x\sqrt{-g} \times [R + d(d-1) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}M^2 A^\mu A_\mu], \tag{5}$$

where $F^{\mu\nu}$ is the rescaled field strength, corresponding to the rescaled massive gauge field A^μ whose mass is M (For more detailed see [32]). The Einstein–Proca equations of motion for metric and gauge field are respectively given by

$$R_{\mu\nu} = -dg_{\mu\nu} + \frac{M^2}{2}A_\mu A_\nu + \frac{1}{2}F^\sigma_\mu F_{\nu\sigma} + \frac{1}{4(1-d)}F^{\rho\sigma}F_{\rho\sigma}g_{\mu\nu}, \quad \nabla_\mu F^{\mu\nu} = M^2 A^\nu. \tag{6a}$$

Then the following metric, along with a gauge field introduced in (A.17a), give a solution of above equations of motion (for more details see appendix A)

$$ds_f^2 = 2(1+\epsilon^2 \ln r)dvdr - r^2[1 + 2\epsilon^2(\ln r - \frac{1}{4}) - \epsilon^2\frac{I_f}{r^3}]dv^2 + r^2(dx_1^2 + dx_2^2) + O(\epsilon^4), \tag{7}$$

whose event horizon will be located at $r = r_h$ given by the largest solution of the following equation

$$1 + 2\epsilon^2(\ln r_h - \frac{1}{4}) - \epsilon^2\frac{I_f}{r_h^3} = 0. \tag{8}$$

Notice that $z = 1 + \epsilon^2$ where $\epsilon \ll 1$ and I_f is given by (A.19) for $\nu \to \infty$. In [29] two specific quench profiles, as a probe of the