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Derivation of spontaneously broken gauge symmetry from the consistency of effective field theory I: Massive vector bosons coupled to a scalar field

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ABSTRACT

We revisit the problem of deriving local gauge invariance with spontaneous symmetry breaking in the context of an effective field theory. Previous derivations were based on the condition of tree-order unitarity. However, the modern point of view considers the Standard Model as the leading order approximation to an effective field theory. As tree-order unitarity is in any case violated by higher-order terms in an effective field theory, it is instructive to investigate a formalism which can be also applied to analyze higher-order interactions. In the current work we consider an effective field theory of massive vector bosons interacting with a massive scalar field. We impose the conditions of generating the right number of constraints for systems with spin-one particles and perturbative renormalizability as well as the separation of scales at one-loop order. We find that the above conditions impose severe restrictions on the coupling constants of the interaction terms. Except for the strengths of the self-interactions of the scalar field, that can not be determined at this order from the analysis of three- and four-point functions, we recover the gauge-invariant Lagrangian with spontaneous symmetry breaking taken in the unitary gauge as the leading order approximation to an effective field theory. We also outline the additional work that is required to finish this program.

1. Introduction

The standard model (SM) is widely accepted as the established consistent theory of the strong, electromagnetic and weak interactions [1]. Invariance under Lorentz and local gauge $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ transformations is taken as the underlying symmetry of the SM. Despite the tremendous success of the SM its structure leaves some unanswered questions. In particular, the electromagnetic and gravitational forces are long-ranged and therefore if they are indeed mediated by massless photons and gravitons, then the corresponding local Lorentz-invariant quantum field theories must be gauge theories [1]. On the other hand, as the weak interaction is mediated by massive particles, one might wonder why it should be described by a gauge theory with the spontaneous symmetry breaking? A gauge-invariant theory with the spontaneous symmetry breaking has been derived by demanding tree-order unitarity of the S-matrix in Refs. [2–5]. This result could be considered as a (more or less) satisfactory answer to the above raised question, however, the modern point of view considers the SM as an effective field theory (EFT) [1] which inevitably violates the tree-order unitarity condition at sufficiently high energies. This motivates us to revisit the problem.

In the current work we address the issue of deriving the most general theory of massive vector bosons by demanding self-consistency in the sense of an EFT. The Lagrangian of an EFT consists of an infinite number of terms, however, the contributions of non-renormalizable interactions in physical quantities are suppressed for energies much lower than some large scale. Renormalizability in the sense of a fundamental theory is replaced by the renormalizability in the sense of an EFT, i.e. that all divergences can be absorbed by renormalizing an infinite number of parameters of the effective Lagrangian. Notice that the condition of perturbative renormalizability in the sense of EFT is not equivalent to the condition of tree-order unitarity. While the tree-order

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unitarity implies renormalizability in the traditional sense, perturbative renormalizability in the sense of EFT is a much weaker condition and it does not imply tree-order unitarity. On the other hand for an EFT to be "effective" it is crucial that the scales are separated, i.e. the contributions of higher order operators in physical quantities are suppressed by powers of some large scale. This condition is much more restrictive than just renormalizability in the sense of EFT. Renormalizability alone can be achieved without introducing scalars, i.e. considering a theory of massive vector bosons and fermions [6]. However, in such theory divergences generated from the leading order Lagrangian are removed by renormalizing the parameters of higher order interactions. This leaves the scales of the renormalized couplings of the higher order terms much too low to explain the tremendous success of the SM. Therefore, in what follows we analyze the constraint structure and the conditions of perturbative renormalizability and scale separation for the most general Lorentz-invariant effective Lagrangian of massive vector bosons interacting with a scalar field. The performed analysis is similar to that of Refs. [7–9] but here we do not assume parity conservation.

The most general Lorentz-invariant effective Lagrangian contains an infinite number of interaction terms. It is assumed that all coupling constants of "non-renormalizable" interactions, i.e. terms with couplings of negative mass-dimensions, are suppressed by powers of some large scale. Massive vector bosons are spin-one particles and therefore they are described by Lagrangians with constraints. To have a system with the right number of degrees of freedom, the coupling constants of the Lagrangian have to satisfy some non-trivial relations. Additional consistency conditions are imposed on the couplings by demanding perturbative renormalizability in the sense of EFT and the separation of scales. Restrictions on the couplings appear because while all loop diagrams can be made finite in any quantum field theory if we include an infinite number of counter terms in the Lagrangian, it is by no means guaranteed that these counter terms are consistent with constraints of the theory of spin-one particles and that the scale separation is not violated.

The paper is organized as follows: In section 2 we specify the assumptions and conditions imposed on the effective Lagrangian. In section 3 we give the effective Lagrangian and carry out the analysis of the constraints. The conditions of perturbative renormalizability and scale separation are obtained in section 4. We summarize and discuss the obtained results in section 5.

2. Starting assumptions and required constraints

The aim of the current work is to construct the most general consistent Lorentz-invariant EFT Lagrangian of three[1] interacting massive vector bosons and a scalar. The free massive vector bosons are described by the Proca Lagrangian which incorporates the second class constraints such that the right number of independent dynamical degrees of freedom are left (three coordinates for each particle). To have a consistent theory of interacting massive vector bosons, the pertinent interaction terms have to be consistent with the second class constraints. This generates some non-trivial relations between the coupling constants of the interaction terms of the most general Lorentz-invariant Lagrangian of a scalar and vector bosons. The next condition we impose is the renormalizability in the sense of an EFT, i.e. that all divergences can be absorbed by renormalizing an infinite number of parameters of the effective Lagrangian. As (some of) the couplings of the effective Lagrangian are already related due to the second class constraints, the condition of perturbative renormalizability cannot be satisfied unless the coupling constants satisfy further restricting conditions. Further constraints on our EFT Lagrangian are imposed by the following considerations: in an EFT, in which the divergences generated by the leading order Lagrangian are removed by renormalizing the parameters of higher order interactions, the scale of renormalized couplings of higher order interaction terms is set by the mass of the vector bosons. The SM is considered to be the leading order approximation to an EFT. One expects that in this EFT the contributions of higher order operators in physical quantities are suppressed by powers of some large scale. The value of this large scale is determined by new physics, that is physics beyond the SM. This leads us to the next condition imposed on our EFT – the separation of scales. That is, we demand that the divergences of the loop diagrams contributing to *physical* scattering amplitudes generated by the leading order Lagrangian should be removable by renormalizing the parameters of the leading order Lagrangian. This condition is much more restrictive than just renormalizability in the sense of EFT. It is actually equivalent to demanding renormalizability of the leading order EFT Lagrangian in the traditional sense, however, not for off-shell Green functions but for the on-shell S-matrix. Notice here that perturbative renormalizability in the sense of EFT in general does not lead to tree-order unitarity. Indeed, renormalizability alone can be achieved without introducing scalars, i.e. considering an EFT of massive vector bosons (and fermions) [6]. However, as is well known, massive Yang–Mills theory is not renormalizable in the traditional sense and it violates tree-order unitarity condition.

To illustrate the problem with the scale separation when divergences are removed by renormalizing the couplings of higher-order operators let us consider an EFT specified by the following Lagrangian

$$\mathcal{L} = -\frac{1}{4}\, F^a_{\mu\nu} F^{a\mu\nu} + \frac{M^2}{2}\, W^a_\mu W^{a\mu} + \mathcal{L}_{\rm ho}, \tag{1}$$

where W^a_μ is the triplet of SU(2) vector bosons, $F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$ is the corresponding field strength tensor and $\mathcal{L}_{\rm ho}$ contains all possible local terms with coupling constants of inverse mass dimensions which are invariant under local $SU(2)$ gauge transformations. The quantum field theory specified by the Lagrangian of Eq. (1) is perturbatively renormalizable in the sense of EFT, i.e. all divergences can be absorbed in the redefinition of fields and an infinite number of coupling constants [10]. It is well known that massive Yang–Mills theory is perturbatively non-renormalizable. Therefore to get rid of the divergences of loop diagrams generated by interaction terms with dimensionless coupling constants, contained in the first term in Eq. (1), one needs to renormalize the couplings of $\mathcal{L}_{\rm ho}$, i.e. couplings with inverse mass dimensions. This has the consequence that even if these couplings are suppressed by some scale much larger than the mass of the vector boson – M for some fixed renormalization condition, slight changes of the renormalization scale will lead to renormalized couplings suppressed only by powers of M divided by some power of the dimensionless coupling g. To be more specific let us consider an example of the vector boson self-energy. Calculating divergent parts of two one-loop diagrams generated by interactions with dimensionless coupling g we obtain

$$\Sigma^{ab,\mu\nu}_{\rm div}(p) = \frac{g^2\delta_{ab}}{96\pi^2 M^4 (n-4)}\left[84M^4 - 14M^2p^2 - p^4\right] \times \left(p^\mu p^\nu - p^2 g^{\mu\nu}\right), \tag{2}$$

[1] The number of massive vector bosons is taken as an input here.