Annals of Physics 397 (2018) 1–64

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Exponential number of equilibria and depinning threshold for a directed polymer in a random potential

Yan V. Fyodorov [a], Pierre Le Doussal [b], Alberto Rosso [c], Christophe Texier [c,*]

[a] King's College London, Department of Mathematics, London WC2R 2LS, United Kingdom
[b] CNRS-Laboratoire de Physique Théorique de l'École Normale Supérieure, 24 rue Lhomond, 75231 Paris, France
[c] LPTMS, CNRS, Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France

A R T I C L E I N F O

Article history:
Received 30 January 2018
Accepted 24 July 2018
Available online 27 July 2018

Keywords:
Directed polymer in random medium
Pinning
Random Schrödinger operator
Anderson localization
Generalized Lyapunov exponent

A B S T R A C T

By extending the Kac–Rice approach to manifolds of finite internal dimension, we show that the mean number $\langle \mathcal{N}_{\text{tot}} \rangle$ of all possible equilibria (i.e. force-free configurations, a.k.a. equilibrium points) of an elastic line (directed polymer), confined in a harmonic well and submitted to a quenched random Gaussian potential in dimension $d = 1 + 1$, grows exponentially $\langle \mathcal{N}_{\text{tot}} \rangle \sim \exp(r\,L)$ with its length L. The growth rate r is found to be directly related to the generalized Lyapunov exponent (GLE) which is a moment-generating function characterizing the large-deviation type fluctuations of the solution to the initial value problem associated with the random Schrödinger operator of the 1D Anderson localization problem. For strong confinement, the rate r is small and given by a non-perturbative (instanton, Lifshitz tail-like) contribution to GLE. For weak confinement, the rate r is found to be proportional to the inverse Larkin length of the pinning theory. As an application, identifying the depinning with a landscape "topology trivialization" phenomenon, we obtain an upper bound for the depinning threshold f_c, in the presence of an applied force, for elastic lines and d-dimensional manifolds, expressed through the mean modulus of the spectral determinant of the Laplace operators with a random potential. We also discuss the question of counting of stable equilibria. Finally, we extend the method to calculate the

* Corresponding author.
E-mail addresses: yan.fyodorov@kcl.ac.uk (Y.V. Fyodorov), ledou@lpt.ens.fr (P. Le Doussal), alberto.rosso@u-psud.fr (A. Rosso), christophe.texier@u-psud.fr (C. Texier).

https://doi.org/10.1016/j.aop.2018.07.029
0003-4916/© 2018 Elsevier Inc. All rights reserved.

asymptotic number of equilibria at fixed energy (elastic, potential and total), and obtain the (annealed) distribution of the energy density over these equilibria (i.e. force-free configurations). Some connections with the Larkin model are also established.
© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Various aspects of the behaviour of a directed polymer, i.e. an elastic line, in a quenched random potential keep attracting permanent research efforts of both physicists and mathematicians for more than three decades. Among other applications, it was at the centre of attention as a model for vortex lines in superconductors, leading to important developments in the physics of pinning (see Refs. [1,2] for reviews). Its connection to the Kardar–Parisi–Zhang growth (see Ref. [3] for review of earlier works) led to a recent outburst of interest, and it was shown that the probability density of the free energy for a long polymer converges to the famous Tracy–Widom distribution [4–8], extending the result for the ground state energy [9].

In this article we address a somewhat different aspect and consider the problem of counting the total number of *equilibria* for a directed polymer (DP), in general harmonically confined, immersed in a random potential. Those are defined as the stationary points (minima, maxima, or saddles) of an energy functional (see below). From a broader perspective, describing the statistical structure of the stationary points of random landscapes and fields of various types is a rich problem of intrinsic current interest in various areas of pure and applied mathematics [10–17]. It also keeps attracting steady interest in the theoretical physics community, and this over more than fifty years [18–25], with recent applications to statistical physics [24–28], neural networks and complex dynamics [17,29,30], string theory [31,32] and cosmology [33].

Note, however, that all the previous works considered only the case of zero internal dimension, equivalent to dealing with a single-particle embedded in a random potential of arbitrary dimension. In such a setting the counting of equilibria (a.k.a, stationary points or force-free configurations) can be placed in a framework of the standard Random Matrix Theory, see Refs. [11,34]. In contrast we aim here to address the counting problem of manifolds of finite internal dimension. As was already anticipated in Ref. [34], in the latter case the problem turns out to be intimately related to properties of random Schrödinger operators appearing in the problems of Anderson localization. To the best of our knowledge, this aspect of the counting problem was never investigated before. Its treatment calls for a quite different technique and requires understanding of less studied properties of random Schrödinger operators, such as the modulus of its determinant and generalized Lyapunov exponents. We develop the corresponding approaches, mainly for the 1D case, in the present article.

2. Model and main results

2.1. The continuous model

We consider the following energy functional

$$\mathcal{H}[u(\tau)] = \int_0^L \mathrm{d}\tau \left[\frac{\kappa}{2} \left(\frac{\partial u(\tau)}{\partial \tau} \right)^2 + \frac{m^2}{2} u^2(\tau) + V(u(\tau), \tau) \right] \tag{1}$$

where $u(\tau),\ \tau \in [0, L]$ describes the polymer configuration trajectory and $\kappa \geqslant 0$ is the elastic energy coefficient (cf. Fig. 1). Unless stated otherwise, in the main text of the paper we assume the fixed ends configuration $u(0) = u(L) = 0$ for simplicity, other types of boundary conditions are briefly discussed in the Appendix A. The random potential $V(u(\tau), \tau)$ is chosen to be Gaussian with zero mean and with a translationally-invariant covariance

$$\left\langle V(u, \tau) V(u', \tau') \right\rangle = \delta(\tau - \tau') R(u - u') , \tag{2}$$

Download English Version:

https://daneshyari.com/en/article/10136821

Download Persian Version:

https://daneshyari.com/article/10136821

Daneshyari.com