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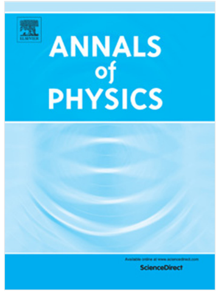


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# Coherence of quantum channels

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We investigate the coherence of quantum channels using the Choi-Jamiołkowski isomorphism. The relation between the coherence and the purity of the channel respects a duality relation. It characterizes the allowed values of coherence when the channel has certain purity. This duality has been depicted via the Coherence-Purity (Co-Pu) diagrams. In particular, we study the quantum coherence of the unital and non-unital qubit channels and find out the allowed region of coherence for a fixed purity. We also study coherence of different incoherent channels, namely, incoherent operation (IO), strictly incoherent operation (SIO), physical incoherent operation (PIO) etc. Interestingly, we find that the allowed region for different incoherent operations maintain the relation $PIO \subset SIO \subset IO$. In fact, we find that if PIOs are coherence preserving operations (CPO), its coherence is zero otherwise it has unit coherence and unit purity. Interestingly, different kinds of qubit channels can be distinguished using the Co-Pu diagram. The unital channels generally do not create coherence whereas some nonunital can. All coherence breaking channels are shown to have zero coherence, whereas, this is not usually true for entanglement breaking channels. It turns out that the coherence preserving qubit channels have unit coherence. Although the coherence of the Choi matrix of the incoherent channels might have finite values, its subsystem contains no coherence. This indicates that the incoherent channels can either be unital or nonunital under some conditions.

## I. INTRODUCTION

Quantum coherence and entanglement are two fundamental resources in quantum information and computation [1–4]. These two resources are closely related [5, 6]. While the concept of entanglement requires at least two particles, coherence can be defined for a single system. Recent developments show that coherence in a quantum system can be a useful resource in quantum algorithm [7–11], quantum meteorology [12], quantum thermodynamics [13–19], and quantum biology [20–22]. Therefore, the study of resource theory of quantum coherence is of immense importance [23–53].

The quantum coherence like other quantum resources is also fragile in the presence of noisy environment. The interaction of quantum systems with environment have been extensively studied using different models – in particular using noisy channels [54]. Characterizing all these channels and their effect on various physical resources are vital [54, 55]. These channels are also important to construct resource theoretic aspect of coherence. Here, in this work, we ask a reverse question. Can we associate coherence with a quantum channel? We answer this question positively. We define the coherence of quantum channels using the Choi-Jamiołkowski (C-J) isomorphism [56, 57]. In this paper, we consider the unital as well as non-unital qubit channels [55, 58, 59]. We compute their coherence and purity analytically. While coherence of a non-unital channel can go up to $\sqrt{2}$ as measured by the $l_1$-norm, the coherence of unital channels can never exceed 1. Using the coherence-purity (CoPu) diagrams, we find that it may be possible to distinguish unital channels and non-unital channels.

The resource theory of coherence require two important elements – free states and free operations [3, 23]. Free states are those which have no coherence in a given reference basis. Free operations do not create any coherence and are known as incoherent operations. Depending on the restrictions (physical requirement), there exist different types of incoherent operations. The largest set of incoherent operations contains Maximally Incoherent Operations (MIO) [60]. The other candidates are Incoherent Operations (IO) [23], Strictly Incoherent Operations (SIO) [4, 24], Physical Incoherent Operations (PIO) [28] etc. There are many other free operations in the literature like Fully Incoherent Operations (FIO), Genuine Incoherent Operations (GIO) etc. It is an important task to understand these operations and distinguish them. In this work, we aim to distinguish these operations using CoPu diagrams. A PIO is in fact a strange candidate – some PIOs which are not Coherence Preserving Operations (CPO) [38] have coherence zero but otherwise it has unit purity and unit coherence. We have constructed all possible FIOs for qubit case and show that they have zero coherence when only row elements are nonzero in their Kraus representation. The FIOs which are diagonal are called GIOs [61]. The FIOs with anti-diagonal elements in their Kraus representation have same CoPu diagrams as GIOs.

We also consider the class of coherence non-generating qubit channels (CNC) as well as the channels to create maximal coherence (CMC). CNC is the bigger set in comparison to all incoherent operations [62]. We also consider other known qubit channels like the class of Pauli channels, degradable and anti-degradable channels, amplitude damping channels, depolarizing channels, and homogenization channels and show that they might be distinguished using CoPu diagrams. Following are the salient features of our results which we address extensively in the main text.

- The coherence of quantum channels has been identified with the coherence of the Choi matrix. The characterization of coherence quantifies the quantumness of the channels, i.e., it might be considered as the quantity

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