Annals of Physics 397 (2018) 365–399

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

A unified approach to Plasmon–Polariton and Brewster mode dynamics in media with interfacial surface-admittance

David C. Christie *, Robin W. Tucker

Physics Department, Lancaster University Lancaster, LA1 4YB, United Kingdom
The Cockcroft Institute, Sci-Tech Daresbury, Daresbury, WA4 4AD, United Kingdom

A R T I C L E I N F O

Article history:
Received 30 April 2018
Accepted 16 August 2018
Available online 31 August 2018

Keywords:
Surface plasmon–polariton
Graphene
Brewster
Meta-surface
Inhomogeneous plane wave
Solar power

A B S T R A C T

In this article we develop an ab-initio formulation for the analysis of certain solutions to the macroscopic Maxwell equations in bulk homogeneous materials that are separated by a planar interface that may sustain field-dependent electric currents induced by a surface admittance tensor. Based on Clemmow's description of complex inhomogeneous plane time-harmonic waves, a systematic procedure is given that yields particular global solutions in terms of solutions to a system of bivariate polynomial and linear eigen-value equations for a set of six complex dimensionless scalars. This algebraic system is amenable to rapid numerical analysis using Maple or Matlab on a laptop. From the resulting solutions, analytic formulae for the electromagnetic fields can be expressed in terms of values for these six scalars and the electromagnetic phenomenological properties of the bulk media and their interface.

We show explicitly how general properties of Surface Plasmon–Polariton (SPP) and Brewster modes follow from this unified viewpoint without appeal to any reflections in a Fresnel formulation. In particular we emphasise that endowing the material interface with (in general) anisotropic admittance properties can lead to new global electromagnetic field characteristics offering new possibilities for the control of surface characteristics by using recent advances in the fabrication of meta-materials.

We illustrate our results by showing how a mono-layer of graphene in a normal magneto-static field can be used to construct a tunable meta-surface for SPP generation and how a simple conducting Ohmic interface can be used to excite both transverse

* Corresponding author at: The Cockcroft Institute, Sci-Tech Daresbury, Daresbury, WA4 4AD, United Kingdom.
E-mail addresses: d.christie@liverpool.ac.uk (D.C. Christie), r.tucker@lancaster.ac.uk (R.W. Tucker).

https://doi.org/10.1016/j.aop.2018.08.008

electric and transverse magnetic Brewster modes as a function of the interface admittance.

1. Introduction

Rapid technological advances in the fabrication and exploitation of artificial materials offer novel possibilities for controlling the behaviour of applied electromagnetic fields. Such possibilities arise since the underlying electronic and molecular properties of matter may give rise to localised field variations in the neighbourhood of regions separating such materials from naturally occurring materials. An analytic mathematical formulation of such matter–field systems becomes feasible if one treats such neighbourhoods as 2-dimensional interfaces across which the bulk electromagnetic constitutive properties and the electromagnetic fields that enter into Maxwell's macroscopic equations exhibit discontinuities. Such constitutive properties are sometimes gleaned directly from experimental data and sometimes derived from condensed matter theory models. Within such a framework one may contemplate the properties of idealised meta-surfaces with controllable constitutive properties as well as fabricated bulk meta-materials.

There exists a vast literature on recent theoretical and technological developments in this field and a somewhat ad-hoc nomenclature has become established for the various field configurations that may be excited in meta-materials. We shall follow these established conventions where possible and also treat the words *interface* and *surface* as synonymous. Electromagnetic excitations induced by surface conduction electrons are referred to as *surface plasmons*. In certain circumstances other excitations may *propagate* in directions parallel to a surface while being attenuated in directions normal to the surface. These are often referred to as *surface Plasmon–Polaritons*. In other situations it may be possible to emulate reflection-less Fresnel refracted plane-fronted waves of *arbitrary polarisation*. We call these *Brewster modes*. Needless to say, the precise definition of all these excitation types depends not only on the constitutive properties of media but also the geometrical properties of the interfaces involved.

Such electromagnetic phenomena have long established valuable applications in Biophotonics including molecular sensing devices, Brewster microscopy for analysing molecular mono-layers and surface enhanced Raman scattering for analysing biological and chemical molecular structures. However the search for an effective *control* of interface excitations in various meta-structures remains an active area of current experimental endeavour [1–6]. The interesting properties of meta-surfaces with a tunable surface impedance have been discussed in [7,8] and the recent discovery of the novel properties of graphene lends new impetus to exploring surface excitations in meta-structures involving this material [9–12] .

In general an analytic approach to solving Maxwell's macroscopic equations for global fields in a composite spatially bounded material containing arbitrarily curved interfaces poses a number of challenging difficulties. Consequently theoretical approaches often rely on simplified models that are bench-marked against various numerical computer codes while descriptions involving Greens functions inevitably lead to various approximation schemes [13].

In this article we offer an alternative approach based on an analytic analysis of Maxwell's macroscopic equations in media with a *single planar* interface using complex plane-fronted field representations. Such representations were pioneered by Clemmow [14] in 1966. In particular we explore solutions for a structure composed of two homogeneous (possibly dispersive) materials separated by a materially homogeneous planar interface that may possess a (possibly dispersive, anisotropic) interfacial admittance tensor. Classes of spatially bounded, time harmonic solutions to this system are sought in any spatial subset of $\mathbb{R}^3$ containing the interface. It is assumed that such a domain forms part of an experimental arrangement in which (possibly collimated) electromagnetic fields can be applied to the system.

Motivated by the potential applications of our methodology to more general interfacial geometries and more general bulk media arrangements we exploit the geometric language of differential forms