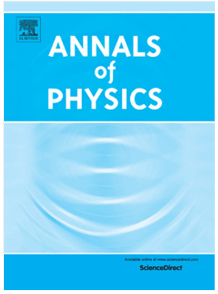

Local gravity theories in conformal superspace

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Local gravity theories in conformal superspace.

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Abstract

The conformal method for the initial value formulation of GR is one of the most powerful and widely used tools in both analytical studies and numerical simulations of solutions. As is well-known, it exploits a hidden spatial conformal symmetry that can be extracted from the equations. But why conformal symmetry? Motivated by these questions and by well-known obstacles to quantum gravity, I look for the most general geometrodynamical symmetries compatible with a reduced physical configuration space for metric gravity. I argue that they are indeed spatial conformal diffeomorphisms. The next question is: what sort of theories most naturally incorporate this symmetry? Demanding locality for an action for metric gravity compatible with these principles determines the allowed operators, both for the purely gravitational part as well as matter couplings. The symmetries guarantee that there are two gravitational propagating physical degrees of freedom, but no explicit refoliation-invariance. The simplest such system has a geometric interpretation as a geodesic model in infinite-dimensional conformal superspace. One example of solution to the equations of motion corresponds to a static Bianchi IX spatial ansatz. The unique coupling to electromagnetism forces the electromagnetic equations to be hyperbolic, enabling us to "build" a standard space-time causal structure. There are, however, deviations from the standard Maxwell equations when space-time anisotropies become too large. Regarding quantization, with the geometric interpretation and the lack of refoliation invariance, the path integral treatment of the symmetries becomes much less involved than the similar approaches to GR. The symmetries form an (infinite-dimensional) Lie algebra, and no BFV treatment is necessary. Moreover, one can use gauge-invariant variational principles *for selecting the boundary conditions* of the path integral. We find that the propagator around the homogeneous solution has up to 6-th order spatial derivatives, giving it plausible regularization properties as in Horava-Lifschitz.

1 Introduction

1.1 The York method and conformal symmetry.

When we want to make predictions about the future behavior of gravitational degrees of freedom, we require an expression of the Einstein equations in terms of evolution equations, i.e. making explicit reference to a time function. Roughly, given an initial value in some space-like surface, these equations should tell us what to expect on some later such surface. This obligates us to break up the original spacetime covariant picture into a foliation of space-time into spatial hypersurfaces (see fig. 1). We don't need to reinvent this account, as it is already standard in the study of general relativity, going by the acronym ADM (after Arnowitt, Deser and Misner [ADM62], see also [Gou07] for a modern review of the methods). Indeed most of the work in numerical GR requires the use of the dynamical approach. Correspondingly, assuming the spacetime manifold is of the form $\Sigma \simeq M \times \mathbb{R}$, we perform a 3+1 split of the spacetime metric:

$$ {}^{(4)}g_{\mu\nu} = \begin{pmatrix} -N^2 + g_{ab}\,\xi^a\,\xi^b & g_{ak}\,\xi^k \\ g_{jk}\,\xi^k & g_{ab} \end{pmatrix}, \tag{1}$$

where we used Greek letters to denote spacetime tensor components, and Latin letters for the spatial ones. The Einstein-Hilbert action is decomposed according to (1),

$$\int d^4x \sqrt{{}^{(4)}g}\; {}^{(4)}R = \int dt d^3x \left(\dot{g}_{ab} p^{ab} + N\,\mathcal{H}[g,p] + \xi^a \mathcal{D}_a[g,p] \right).$$

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