Results in Physics 11 (2018) 65–71

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Unsteady shrinking embedded horizontal sheet subjected to inclined Lorentz force and Joule heating, an analytical solution

Emran Khoshrouye Ghiasi, Reza Saleh*

Department of Mechanical Engineering, College of Engineering, Mashhad Branch, Islamic Azad University, Mashhad, Iran

ARTICLE INFO

Keywords:
Casson fluid
Unsteady
Buongiorno model
Magnetic field
OHAM

ABSTRACT

This article focuses on the 2D flow of an incompressible Casson fluid over an unsteady shrinking horizontal sheet under inclined Lorentz force and Joule heating. The governing partial differential equations (PDEs), which account for the effect of Buongiorno model, are converted into the nonlinear ordinary differential equations (ODEs) through similarity variables. An effective method i.e., optimal homotopy analysis method (OHAM) is employed here to solve the system of presented ODEs. The results are compared and validated with those of numerical findings available in the literature. It is found that the OHAM can provide an effective way to ensure convergence of the series solution. Utilizing this fact, the effect of governing physical parameters on the skin friction coefficient, local Nusselt number and local Sherwood number are thoroughly investigated.

Introduction

Due to the high efficiency of nanofluids in heat and mass transfer, there has recently been a substantial increase in the number of studies concerning the nanofluid dynamics. The nanoparticles are between 1 and 100 nm in size. For this reason, the difficulty lies in the modeling of slip mechanisms which can be easily implemented by the Buongiorno model [1]. This model, on the other hand, can incorporate the effects of thermophoresis and Brownian diffusion as two important slip mechanisms between the phases. According to the recent research studies, Sheremet et al. [2] analyzed both thermophoresis and Brownian diffusion as well as buoyancy force of conducting nanofluids in 2D entrapped triangular cavities using finite difference method (FDM). Their results showed that an increase in the Rayleigh number, as opposed to the average Nusselt number, generates the steady state heat transfer mode for the upper heated surface. Utilizing the homotopy analysis method (HAM), Shehzad et al. [3] studied convective heat transfer in a horizontal symmetric wavy channel based on the Buongiorno model. The study found that the momentum boundary layer thickness decreases with an increase in the Prandtl number in the vicinity of the surface. Malvandi et al. [4] utilized the modified Buongiorno model proposed by Yang et al. [5] to develop convective heat transfer of nanofluids within a vertical annulus. Also, it is to be noted that the buoyancy force, which was not investigated by Yang et al. [5], had been taken into account by Malvandi et al. [4]. Afterwards, Zhu et al. [6] considered Al_2O_3-H_2O and TiO_2-H_2O as two types of nanofluids to analyze heat transfer between two rotating disks and showed that an increase in Brownian motion parameter may account for changes in thermal boundary layer thickness.

In rheology, the pseudoplastic behavior is occurred when the fluid viscosity decreases with an increase in the shear stress. This issue can be observed in the shear thinning fluids [7]. One of the most important time-independent fluids associated with this classification is the Casson model [8] which has extensively been studied during the last decades. In the sixties, Scott Blair [9], Stoltz and Larcan [10] and Deakin [11] presented some biological applications of the aforementioned model. More recently, Abd El-Aziz and Yahya [12] studied the unsteady magnetohydrodynamic (MHD) slip flow of a Casson fluid over an infinite permeable surface and showed that accounting for the effect of Hall current increases the axial and transverse velocity profiles simultaneously. As investigated by Reddy et al. [13], utilizing the ferrofluids e.g., Fe_3O_4 with Casson fluid can enhance heat transfer over an upper surface of a paraboloid of revolution. They also extended the approach proposed by Animasaun and Sandeep [14] for nonlinear thermal radiation and viscous dissipation of ferrofluids. Walicka and Falicki [15] presented an electrorheological Casson model as a function of Reynolds number which can predict the pressure distribution between two parallel disks and concentric spherical surfaces. They showed that the pressure decreases with an increase in Reynolds number. Pal et al. [16] took the influence of Joule dissipation and magnetic field as well as thermal radiation into account to investigate the flow of a Casson fluid past a vertical stretching sheet, and found that

* Corresponding author.
E-mail address: r-saleh@mshdiau.ac.ir (R. Saleh).

https://doi.org/10.1016/j.rinp.2018.07.026
Received 14 March 2018; Received in revised form 27 May 2018; Accepted 21 July 2018
Available online 02 August 2018

using the Runge-Kutta-Fehlberg method together with the shooting method can represent the fairly accurate results. It is worth mentioning that more details of the Casson model are set out in Refs. [17–20].

Although there exists a gap in the Navier-Stokes equation to describe the motion of a non-Newtonian fluid, utilizing these time-independent fluids can provide a nonlinear relationship between the shear stress and strain rate. This article deals with the analytical optimal solution of 2D flow of an incompressible Casson fluid in the vicinity of an unsteady shrinking horizontal sheet under inclined Lorentz force and Joule heating based on the Buongiorno model. It should be noted that there have been no reports of this issue being solved to date.

Governing equations

The rheological equation for the Casson model is given by [8],

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)e_{ij}, \pi > \pi_c, \\ 2\left(\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, \pi < \pi_c, \end{cases} \tag{1}$$

and,

$$p_y = \frac{\mu_B\sqrt{2\pi}}{\lambda}, \tag{2}$$

where τ_{ij} is the Cauchy stress tensor, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of the fluid, e_{ij} is the (i, j)th component(s) of the strain rate, $\pi = e_{ij}e_{ij}$ is the product of the strain rate component(s), π_c is the critical value of this product based on the non-Newtonian fluid and λ is the Casson fluid parameter [8]. Since $\pi > \pi_c$, one can represent [21],

$$\mu = \mu_B + \frac{p_y}{\sqrt{2\pi}}, \tag{3}$$

where μ is the dynamic viscosity of the fluid. By substituting Eq. (2) into Eq. (3), the kinematic viscosity can be derived as [21],

$$\upsilon = \frac{\mu}{\rho} = \frac{\mu_B}{\rho}\left(1 + \frac{1}{\lambda}\right), \tag{4}$$

where ρ is the density.

For the transient 2D flow in the Cartesian coordinate, the velocity, temperature and nanoparticle concentration fields take the form,

$$\mathbb{V} = [u(x, y, t), v(x, y, t)],\ \mathbb{T} = T(x, y, t),\ \mathbb{C} = C(x, y, t), \tag{5}$$

where u and v are the velocity components along the x and y axes, respectively, t is the time, T is the temperature and C is the nanoparticle concentration.

The following system of PDEs that govern the transient 2D flow of a Casson fluid can be written as,

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1 + \frac{1}{\lambda}\right)\frac{\partial^2 u}{\partial y^2} - \underline{\frac{\sigma B_0^2}{\rho}u\sin^2\psi}, \\ \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \varsigma\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \\ \qquad + \underline{\frac{\sigma B_0^2}{\rho c_p}u^2}, \\ \frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}, \end{cases} \tag{6}$$

where σ is the electrical conductivity, B_0 is the magnetic field strength, ψ is the inclination angle of the magnetic field, $\alpha(=\frac{k}{\rho c_p})$ is the thermal diffusivity, $\varsigma(=\frac{(\rho c)_p}{(\rho c)_f})$ is the ratio of effective heat capacity of the nanoparticle to effective heat capacity of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and T_∞ is the ambient temperature. It should be noted that the underlined terms on the right-hand sides of Eq. (6) indicate the presence of inclined Lorentz force and Joule heating, respectively.

The associated initial and boundary conditions are stated as follows,

$$\begin{cases} t < 0: u = 0,\ v = 0,\ T = T_\infty,\ C = C_\infty, \text{ for all } x, y, \\ t \geq 0: u = u_w(x, t) = -\frac{bx}{1-\beta t},\ v = v_w(x, t),\ T = T_w,\ C = C_w, \text{ at } y = 0, \\ u \to 0,\ T \to T_\infty,\ C \to C_\infty, \text{ as} y \to \infty, \end{cases} \tag{7}$$

where C_∞ is the nanoparticle concentration far from the sheet, $u_w(x, t)$ is the velocity of unsteady shrinking sheet, $v_w(x, t)$ is the rate of mass transfer, T_w and C_w are the temperature and nanoparticle concentration of the sheet, respectively, $b > 0$ is a constant with dimension $(\text{time})^{-1}$ and β is a parameter that shows unsteadiness of the problem.

To derive the similarity solution of Eq. (6), the following variables can be taken as,

$$\zeta = \sqrt{\frac{b}{\upsilon(1-\beta t)}}y,\ \varphi = \sqrt{\frac{b\upsilon}{1-\beta t}}xf(\zeta),\ \theta(\zeta) = \frac{T-T_\infty}{T_w-T_\infty},\ \phi(\zeta) = \frac{C-C_\infty}{C_w-C_\infty}, \tag{8}$$

where ζ is the similarity parameter, φ is the stream function which satisfies the equation of continuity i.e., $(u, v) = \left(\frac{\partial\varphi}{\partial y}, -\frac{\partial\varphi}{\partial x}\right)$, $\theta(\zeta)$ is the dimensionless temperature, $\phi(\zeta)$ is the dimensionless nanoparticle concentration and $f(\zeta)$ is an ordinary function involved in the stream function. Herein, one can establish as follows,

$$u = \frac{bx}{1-\beta t}\frac{\partial f(\zeta)}{\partial\zeta},\ v = -\sqrt{\frac{b\upsilon}{1-\beta t}}f(\zeta). \tag{9}$$

By substituting Eq. (9) into Eq. (6), the following system of ODEs can be stated as,

$$\begin{cases} \left(1 + \frac{1}{\lambda}\right)\frac{\partial^3 f}{\partial\zeta^3} + f\frac{\partial^2 f}{\partial\zeta^2} - \bar{b}\left(\frac{\partial f}{\partial\zeta} + \frac{1}{2}\zeta\frac{\partial^2 f}{\partial\zeta^2}\right) - \frac{\partial f}{\partial\zeta}\left(\frac{\partial f}{\partial\zeta} + \text{Ha}^2\sin^2\psi\right) = 0, \\ \frac{1}{\Pr}\frac{\partial^2\theta}{\partial\zeta^2} + \frac{\partial\theta}{\partial\zeta}\left(f - \frac{1}{2}\bar{b}\zeta + Nb\frac{\partial\phi}{\partial\zeta}\right) + Nt\left(\frac{\partial\theta}{\partial\zeta}\right)^2 + \text{Ha}^2 Ec\left(\frac{\partial f}{\partial\zeta}\right)^2 = 0, \\ \frac{\partial^2\phi}{\partial\zeta^2} + Le\left(f\frac{\partial\phi}{\partial\zeta} - \frac{1}{2}\bar{b}\zeta\frac{\partial\phi}{\partial\zeta}\right) + \frac{Nt}{Nb}\frac{\partial^2\theta}{\partial\zeta^2} = 0, \end{cases} \tag{10}$$

where $\bar{b} = \frac{\beta}{b}$ is the unsteadiness parameter, $\text{Ha}^2 = \frac{\sigma B_0^2}{\rho b}$ is the Hartmann number, $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $Nb = \frac{\varsigma D_B}{\upsilon}(C_w - C_\infty)$ is the Brownian motion parameter, $Nt = \frac{\varsigma D_T}{\upsilon T_\infty}(T_w - T_\infty)$ is the thermophoresis parameter, $Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}$ is the Eckert number and $Le = \frac{\upsilon}{D_B}$ is the Lewis number. The appropriate boundary conditions are given by,

$$\begin{cases} f(\zeta) = A,\ \frac{\partial f(\zeta)}{\partial\zeta} = -1,\ \theta(\zeta) = 1,\ \phi(\zeta) = 1, \text{ at } \zeta = 0, \\ \frac{\partial f(\zeta)}{\partial\zeta} \to 0,\ \theta(\zeta) \to 0,\ \phi(\zeta) \to 0, \text{ as } \zeta \to \infty, \end{cases} \tag{11}$$

where A is the mass suction parameter.

For the sake of practical interest, the skin friction coefficient, local Nusselt number and local Sherwood number can be expressed as,

$$C_{fx} = \frac{\tau_w}{\rho u_w^2},\ Nu_x = \frac{xq_w}{k(T_w - T_\infty)},\ Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \tag{12}$$