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An analytical method for free vibration of multi cracked and stepped nonlocal nanobeams based on wave approach

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ABSTRACT

Various discontinuities and boundary conditions affect natural frequencies of nanobeams. In this paper, free lateral vibration of Euler-Bernoulli nanobeam with multiple discontinuities is studied. The governing equations are developed by using Eringen's nonlocal elasticity theory. Cracks and steps are considered as discontinuities. Based on wave approach, vibrations take into account as moving waves along the structure. Waves propagate through waveguide and are reflected and transmitted at the discontinuities and boundaries. The propagation, reflection and transmission matrices are derived for discontinuities and boundaries. Cracks are modelled by a massless torsional spring with infinitesimal length and steps are considered as two connected nanobeams with different cross-sectional areas and mechanical properties. Boundary is formed by combining a torsional and a translational spring. Also buckyball (as a lumped mass) at the tip of the beam is considered as a boundary condition. Using propagation and appropriated reflection and transmission matrices for each case, leads to an analytical comprehensive succinct method so-called wave approach to analyse the free lateral vibration of nanobeams. The flexural natural frequencies are derived analytically for cracked or/and stepped nanobeams with ordinary boundary conditions or buckyball at the tip. The effects of crack severity, changing ratio of cross-sectional area as step, cracks and steps location, mass of buckyball and small-scale parameter on natural frequencies are deliberated. This approach is demonstrated with a number of examples that can be used as benchmarks for other works. The results are compared with other methods.

Introduction

Iijima's [1] milestone paper on carbon nanotubes (CNTs) opened a new season of nanotechnology developments. Nowadays, multiplicity capacities and capabilities of nanoelectromechanical system (NEMS) devices can be found in a number of research publications [2–5]. Eringen [6,7], introduced the nonlocal elasticity theory stating that the strain field at all points of the substance affects the stress tensor at each specific point. Numerous research have used the nonlocal elasticity theory to derive the governing equations. The results for wave propagation (axial, torsional and lateral), buckling and vibration from molecular mechanics/dynamics and lattice dynamics, are compared with the results from nonlocal continuum theory. There is a good agreement between nonlocal continuum modelling and molecular dynamic simulations [8–10]. This endorsement on the results, encourages researchers to utilize Eringen's theory for future studies. By using nonlocal elasticity deflection of cantilever nanobeams [11], lateral vibration of Euler-Bernoulli nanobeam [12], buckling and vibration of nanotubes [13], flexural vibration of Timoshenko beam [14] and buckling, bending and vibration of nanobeams [15,16] have been studied. Thai [17] suggested a nonlocal shear deformation theory based on constitutive relations of Eringen's theory in differential form for bending, buckling, and vibration of nanobeams. Free lateral vibration of double-walled nanobeam is investigated by Zhang et al. [18] and, Murmu and Adhikari [19] via nonlocal elasticity. Also, nonlinear vibration of embedded multi-walled Timoshenko nanobeams based on Eringen's theory have been studied [20,21]. Vibration analysis of functionally Graded (FG) nanostructures [22,23] and behaviour of bio-nanosystems [24] are also of interest for researchers. Wang [25] studied wave propagation in CNTs and extracted dispersion relation for Euler-Bernoulli and Timoshenko nanobeams. Narendar and Gopalakrishnan [26] presented the effect of small-scale on wavenumber and wave speeds of nanorods by explicit expressions. In another work, Narendar [27] added the effects of lateral inertia on wave propagation in nanorods. Narendar and Gopalakrishnan [28] studied wave propagation in rotating nanotube. Aydogdu [29] investigated the axial wave propagation in multiwalled carbon nanotubes. The effects of nonlocal parameters and Van der Waals forces are considered in his work. He also derived explicit

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expressions for wave speeds.

Computational methods such as Differential Quadrature Method (DQM) [30–34], Fourier solution [35], Generalized Differential Quadrature (GDQ) [36], Navier's Method [37], Finite Element Method (FEM) [38,39], Wentzel–Kramers–Brillouin (WKB) method [40,41] and the Galerkin method [42–44] are utilized to solve the nonlocal governing equations of nanostructures.

Steps and cracks are the most common cited defects in nanostructures. However, they may be formed intentionally to achieve the desired frequencies of nanostructures. Steps are considered as abrupt changes in cross-sectional area such as CNT heterojunctions or two connected nanobeam portions with different material properties [45,38]. Crack(s) can be shaped during production process for various reasons. Thermal expansion through heating, causes to create crack(s) in ZnO nanowire and nanobelt [46,47]. Cracks apply local flexibility and stiffness in structure. With regards to the importance of the subject of crack effects on mechanical behaviours of nanostructures, a significant percentage of the research have been dedicated to this field [48–55]. In lateral vibration analysis, the crack is modelled as an equivalent rotational spring which is placed at the crack location [50–52]. This massless spring with infinitesimal length, connects two nanobeam segments in each side of the crack. The same approach is used with translational spring for axial vibration [53,54] and rotational spring for torsional vibration [55]. Roostai and Haghpanahi [51] analysed free vibration of multi cracked nanobeams with different boundary conditions. They used common classic boundary conditions to solve the problem and as will be shown, this assumption has a great impact on natural frequencies. Also, by increasing the number of cracks, the solution becomes more complicated progressively. In the present work, steps are considered as discontinuities besides the cracks. Moreover, the nonlocal boundary conditions and also a lumped mass at the tip as a boundary condition are deliberated. Furthermore, increasing the number of discontinuities adds no difficulty in the present approach. Buckminsterfullerene or buckyball is a spherical fullerene which can be attached at the tip of a nanotube [56,57]. In cases of nanosensors/resonators, changes in the mass of buckyballs cause a shift in nanostructure resonant frequencies [58]. This feature in NEMS application, allows the possibility of using the nanobeams as tuneable nanoresonators. Murmu and Adhikari [59] derived a transcendental closed-form equation for axial natural frequencies with arbitrary mass of buckyball. They have shown a high dependency of natural frequencies on the mass of the buckyball. Also, the effect of nonlinearity on mechanical behaviour in nanostructures covers a great part of research [60–62]. Furthermore, it is shown that the effect of surface layer could be as important as the nonlocal effect [63–68]. The effect of surface energy decreases by increasing the thickness of nanostructure [65].

Vibrations can be taken into account as propagating waves in a structure as a waveguide. Many research deal with wave propagation, reflection and transmission relations in solid medium [69–73]. Assembling these relations provides a method for vibration analysis, named as wave approach. Mei used this approach to study free axial, torsional and lateral vibration of classical rods and Euler-Bernoulli beams with lumped masses at the tips [74], and Timoshenko beams [75]. Mei [76] presented an analytical solution for coupled lateral and longitudinal vibrations of portal planar frame and L-shaped structures. She also investigated longitudinal vibration rods with four different rod theories [77]. Mei and Sha [78] applied the wave method to analyse simple spatial structures. For nanostructures, by using this method, Loghmani et al. [79] studied axial vibration of multi cracked and stepped nanorods. Baohui et al. [80] analysed free lateral vibration of nanotube which are conveying fluid. Bahrami and Teimourian investigated free vibrations of Euler-Bernoulli [81] and Timoshenko [82] nanobeams.

The aim of the present work is vibration analysis of multi cracked, stepped nanobeams by using wave approach. The governing equation for lateral vibration of Euler-Bernoulli nanobeam is developed based on Eringen's nonlocal elasticity theory. Expressions for bending moment, shear force, continuity and equilibrium conditions at cracks, steps, general boundary and boundary with lumped mass are derived. Then, these relations are described in wave formulation form and the wave propagation, transmission and reflection matrices are obtained for different discontinuities and boundary conditions. Subsequently, the obtained matrices are combined for free lateral vibration analysis via wave approach to achieve natural frequencies. The approach is applied to several cases. As will be shown, there is no limitation to the number of cracks and steps for the analysis and in each case an algebraic equation is obtained. In some cases, an exact closed-form solution is derived and in other ones the achieved explicit expressions should be solved numerically. The considered boundary conditions in this work are: simply-simply supported (S-S), clamped-clamped (C-C), clamped-simply supported (C-S), clamped-free (C-F) and cantilever with a lumped mass at tip. The effects of crack severity, changing ratio of cross-sectional area as step, crack and step location, mass of buckyball and small-scale parameter on natural frequencies are discussed. In the present work, the effects of multiple discontinuities on natural frequencies are illustrated both generally and each one alone. As will be shown, the presented approach is applicable for more complicated situations. The results are compared with other existing methods.

Governing equations of nanobeam based on nonlocal elasticity

In the nonlocal elasticity theory [6,7], the stress tensor at a reference point depends on the strain field at all points of the medium. Experimental investigations on phonon dispersion and lattice dynamics, have defended this statement. This theory states that the relationship of stress and strain for a homogeneous elastic solid is:

$$\sigma_{ij}(x) = \int_V \varphi(|\boldsymbol{x}-\boldsymbol{x}'|, \eta)\, t_{ij}\, d\,\boldsymbol{V}(x'), \qquad \forall\, x \in \boldsymbol{V} \tag{1}$$

in which σ_{ij} and t_{ij} are the nonlocal and local stress tensors, respectively. The integration covers over the full medium volume V. $\varphi(|\boldsymbol{x}-\boldsymbol{x}'|, \eta)$ is the nonlocal modulus which shows the effect of the strain at the point $\boldsymbol{x}'$ on the stress at the point $\boldsymbol{x}$ and η is the property of the substance which is based on internal and external length characteristic (e.g. length of two carbon molecule bonds and length of nanobeam). Nonlocal constitutive relations for the Euler- Bernoulli nanobeam can be described as [11]:

$$\sigma_{xx}-(e_0 a)^2\frac{\partial^2}{\partial x^2}\sigma_{xx} = E\varepsilon_{xx} \tag{2}$$

where ε_{xx} denotes the normal strain and is given by:

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2} \tag{3}$$

in which x and z are the coordinates of the length and midplane of the beam, respectively and w is the lateral displacement. According to the definition, the bending moment is:

$$M = \int z\sigma dA \tag{4}$$

From Eqs. (2), (3) and (4) the bending moment M can be explained as:

$$M-(e_0 a)^2\frac{\partial^2 M}{\partial x^2} = -EI\frac{\partial^2 w}{\partial x^2} \tag{5}$$

where I presents the second moment of area. Equilibrium conditions for the free lateral vibrating Euler-Bernoulli beam is [18]:

$$Q = \frac{\partial M}{\partial x} \tag{6}$$

$$\frac{\partial Q}{\partial x} = \rho A\frac{\partial^2 w}{\partial t^2} \tag{7}$$